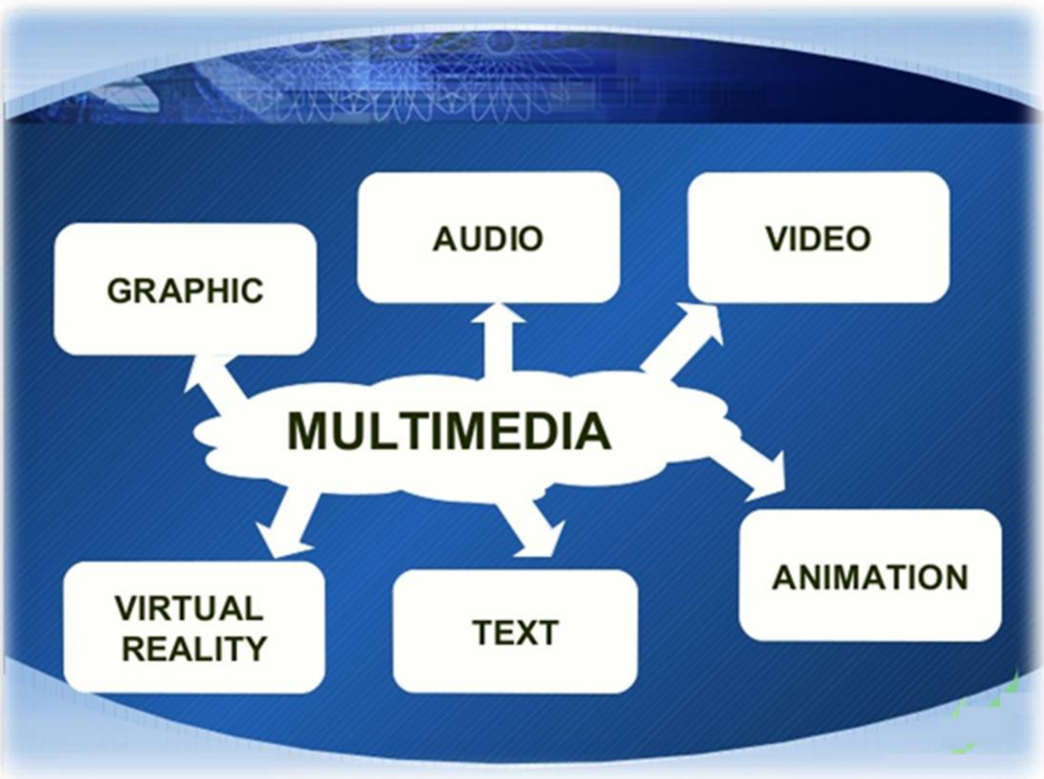

Tikrit University

Computer Science Dept.

Master Degree

Lecture 7

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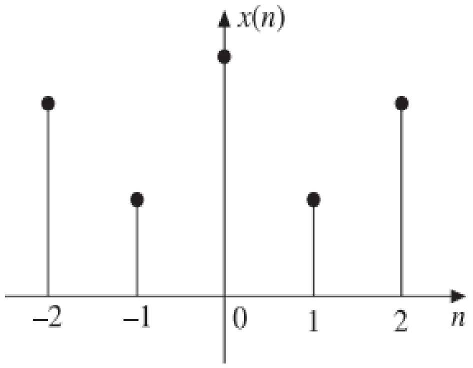
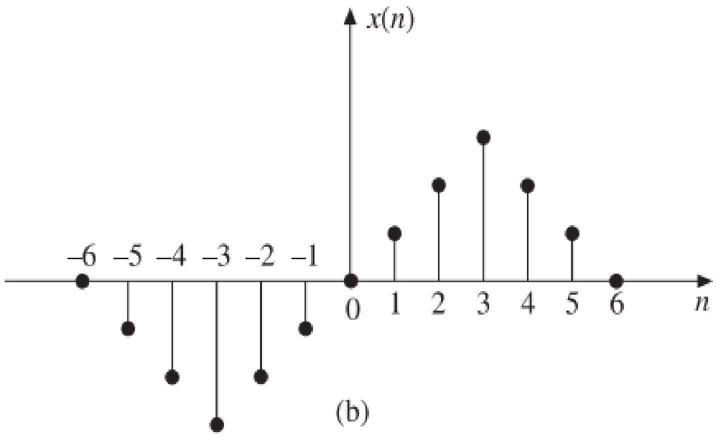
Classification of Signals

The signals can be classified based on their nature and characteristics in the time domain. They are broadly classified as: (i) continuous-time signals and (ii) discrete-time signals. The signals that are defined for every instant of time are known as continuous-time signals. The continuous-time signals are also called analog signals. They are denoted by x(t). They are continuous in amplitude as well as in time. Most of the signals available are continuous-time signals. The signals that are defined only at discrete instants of time are known as discrete-time signals. The discrete-time signals are continuous in amplitude, but discrete in time. For discrete-time signals, the amplitude between two-time instants is just not defined. For discrete-time signals, the independent variable is time n. Since they are defined only at discrete instants of time, they are denoted by a sequence x(nT) or simply by x(n) where n is an integer. Figure 1(a, b) shows the graphical representation of discrete-time signals.



(a)



(b)

Both continuous-time and discrete-time signals are further classified as follows:

1. Deterministic and random signals

2. Periodic and Aperiodic signals

3. Causal and non-causal signals.

4. Energy and power signals

5. Even and odd signals

1. Deterministic and Random Signals

A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called deterministic signal. A deterministic signal can be completely represented by mathematical equation at any time and its nature and amplitude at any time can be predicted.

Examples: Sinusoidal sequence $x(n) = \cos \omega n$, Exponential sequence $x(n) = e^{j\omega n}$, ramp sequence $x(n) = \alpha n$.

A signal characterized by uncertainty about its occurrence is called a non-deterministic or random signal. A random signal cannot be represented by any mathematical equation. The behaviour of such a signal is probabilistic in nature and can be analyzed only stochastically. The pattern of such a signal is quite irregular. Its amplitude and phase at any time instant cannot be predicted in advance. A typical example of a non-deterministic signal is thermal noise.

2. Periodic and Aperiodic Signals

A signal which has a definite pattern and repeats itself at regular intervals of time is called a periodic signal, and a signal which does not repeat at regular intervals of time is called a non-periodic or aperiodic signal. A discrete-time signal $x(n)$ is said to be periodic if it satisfies the condition $x(n) = x(n + N)$ for all integers n. The smallest value of N which satisfies the above condition is known as fundamental period.

If the above condition is not satisfied even for one value of n, then the discrete-time signal is aperiodic. Sometimes aperiodic signals are said to have a period equal to infinity.

The angular frequency is given by

$$\omega = \frac{2\pi}{N}$$

∴ Fundamental period $$N = \frac{2\pi}{\omega}$$

The sum of two discrete-time periodic sequences is always periodic.

Some examples of discrete-time periodic/non-periodic signals are shown in **Figure 2**

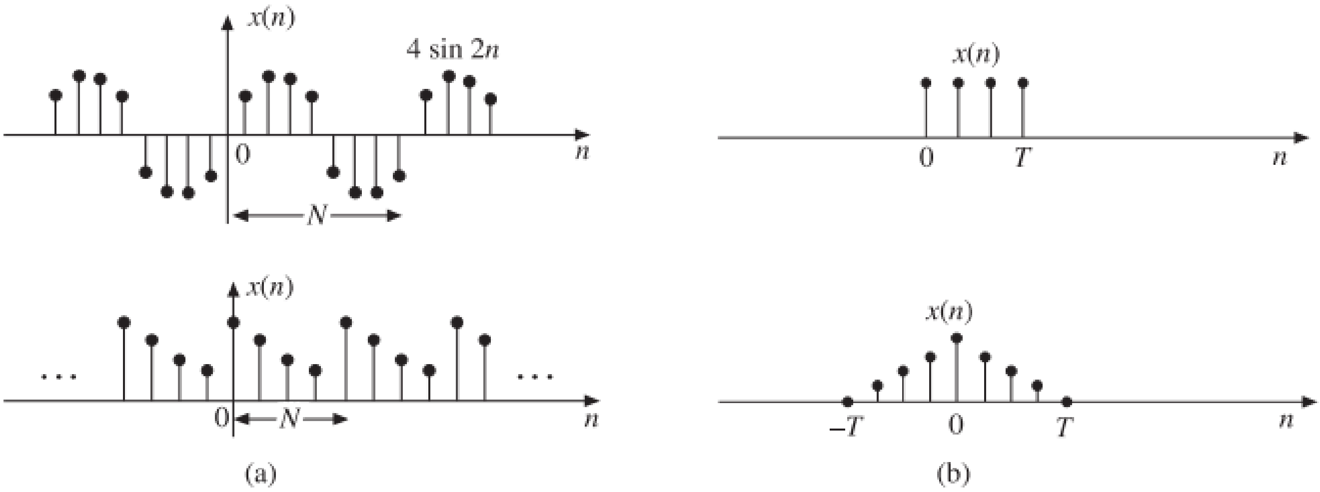


Figure 2 Examples of discrete-time: (a) Periodic and (b) Non-periodic signals.

Example 1 Show that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic only if $\omega_0/2\pi$ is a rational number.

Solution: Given $$x(n) = e^{j\omega_0 n}$$

$x(n)$ will be periodic if $$x(n + N) = x(n)$$

i.e. $$e^{j[\omega_0(n+N)]} = e^{j\omega_0 n}$$

i.e. $$e^{j\omega_0 N}\, e^{j\omega_0 n} = e^{j\omega_0 n}$$

This is possible only if $$e^{j\omega_0 N} = 1$$

This is true only if $$\omega_0 N = 2\pi k$$

where k is an integer.

∴ $$\frac{\omega_0}{2\pi} = \frac{k}{N} \text{ Rational number}$$

This shows that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic if $\omega_0/2\pi$ is a rational number.

Example 2 Obtain the condition for discrete-time sinusoidal signal to be periodic.

Solution: In case of continuous-time signals, all sinusoidal signals are periodic. But in discrete-time case, not all sinusoidal sequences are periodic.

Consider a discrete-time signal given by

$$x(n) = A \sin(\omega_0 n + \theta)$$

where A is amplitude, ω_0 is frequency and θ is phase shift.

A discrete-time signal is periodic if and only if

$$x(n) = x(n + N) \text{ for all } n$$

Now, $$x(n + N) = A \sin[\omega_0(n + N) + \theta)] = A \sin(\omega_0 n + \theta + \omega_0 N)$$

Therefore, $x(n)$ and $x(n + N)$ are equal if $\omega_0 N = 2\pi m$. That is, there must be an integer m such that

$$\omega_0 = \frac{2\pi m}{N} = 2\pi\left[\frac{m}{N}\right]$$

or $$N = 2\pi\left[\frac{m}{\omega_0}\right]$$

From the above equation we find that, for the discrete-time signal to be periodic, the fundamental frequency ω_0 must be a rational multiple of 2π. Otherwise the discrete-time signal is aperiodic. The smallest value of positive integer N, for some integer m, is the fundamental period.

Example 3 Determine whether the following discrete-time signals are periodic or not. If periodic, determine the fundamental period.

(a) $\sin(0.02\pi n)$

(b) $\sin(5\pi n)$

(c) $\cos 4n$

(d) $\sin\frac{2\pi n}{3} + \cos\frac{2\pi n}{5}$

(e) $\cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$

(f) $\cos\left(\frac{\pi}{2} + 0.3n\right)$

(g) $e^{j(\pi/2)n}$

(h) $1 + e^{j2\pi n/3} - e^{j4\pi n/7}$

Solution:

(a) Given $$x(n) = \sin(0.02\pi n)$$

Comparing it with $$x(n) = \sin(2\pi f n)$$

we have $$0.02\pi = 2\pi f \quad \text{or} \quad f = \frac{0.02\pi}{2\pi} = 0.01 = \frac{1}{100} = \frac{k}{N}$$

Here f is expressed as a ratio of two integers with $k = 1$ and $N = 100$. So it is rational. Hence the given signal is periodic with fundamental period $N = 100$.

(b) Given $x(n) = \sin(5\pi n)$

Comparing it with $x(n) = \sin(2\pi fn)$

we have $2\pi f = 5\pi$ or $f = \frac{5}{2} = \frac{k}{N}$

Here f is a ratio of two integers with $k = 5$ and $N = 2$. Hence it is rational. Hence the given signal is periodic with fundamental period $N = 2$.

(c) Given $x(n) = \cos 4n$

Comparing it with $x(n) = \cos 2\pi fn$

we have $2\pi f = 4$ or $f = \frac{2}{\pi}$

Since $f = (2/\pi)$ is not a rational number, $x(n)$ is not periodic.

(d) Given $x(n) = \sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$

Comparing it with $x(n) = \sin 2\pi f_1 n + \cos 2\pi f_2 n$

we have $2\pi f_1 = \frac{2\pi}{3}$ or $f_1 = \frac{1}{3} = \frac{k_1}{N_1}$

$\therefore \quad N_1 = 3$

and $2\pi f_2 n = \frac{2\pi}{5}$ or $f_2 = \frac{1}{5}$

$\therefore \quad N_2 = 5$

Since $\frac{N_1}{N_2} = \frac{3}{5}$ is a ratio of two integers, the sequence $x(n)$ is periodic. The period of $x(n)$ is the LCM of N_1 and N_2. Here LCM of $N_1 = 3$ and $N_2 = 5$ is 15. Therefore, the given sequence is periodic with fundamental period $N = 15$.

(e) Given $x(n) = \cos\left(\frac{n}{6}\right) \cos\left(\frac{n\pi}{6}\right)$

Comparing it with $x(n) = \cos(2\pi f_1 n) \cos(2\pi f_2 n)$

we have $2\pi f_1 n = \frac{n}{6}$ or $f_1 = \frac{1}{12\pi}$

which is not rational.

And $2\pi f_2 n = \frac{n\pi}{6}$ or $f_2 = \frac{1}{12}$

which is rational.

Thus, $\cos(n/6)$ is non-periodic and $\cos(n\pi/6)$ is periodic. $x(n)$ is non-periodic because it is the product of periodic and non-periodic signals.

(f) Given $$x(n) = \cos\left(\frac{\pi}{2} + 0.3n\right)$$

Comparing it with $$x(n) = \cos(2\pi fn + \theta)$$

we have $2\pi fn = 0.3n$ and phase shift $\theta = \dfrac{\pi}{2}$

$\therefore$ $$f = \frac{0.3}{2\pi} = \frac{3}{20\pi}$$

which is not rational.
Hence, the signal $x(n)$ is non-periodic.

(g) Given $$x(n) = e^{j(\pi/2)n}$$

Comparing it with $$x(n) = e^{j2\pi fn}$$

we have $$2\pi f = \frac{\pi}{2} \quad \text{or} \quad f = \frac{1}{4} = \frac{k}{N}$$

which is rational.
Hence, the given signal $x(n)$ is periodic with fundamental period $N = 4$.

(h) Given $$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7}$$

Let $$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7} = x_1(n) + x_2(n) + x_3(n)$$

where $x_1(n) = 1$, $x_2(n) = e^{j2\pi n/3}$ and $x_3(n) = e^{j4\pi n/7}$

$x_1(n) = 1$ is a d.c. signal with an arbitrary period $N_1 = 1$

$$x_2(n) = e^{j2\pi n/3} = e^{j2\pi f_2 n}$$

$\therefore$ $$\frac{2\pi n}{3} = 2\pi f_2 n \quad \text{or} \quad f_2 = \frac{1}{3} = \frac{k_2}{N_2} \quad \text{where } N_2 = 3$$

Hence $x_2(n)$ is periodic with period $N_2 = 3$.

$$x_3(n) = e^{j4\pi n/7} = e^{j2\pi f_3 n}$$

$\therefore$ $$\frac{4\pi n}{7} = 2\pi f_3 n \quad \text{or} \quad f_3 = \frac{2}{7} = \frac{k_3}{N_3} \quad \text{where } N_3 = \frac{7}{2}$$

Now, $$\frac{N_1}{N_2} = \frac{1}{3} = \text{Rational number}$$

$$\frac{N_1}{N_3} = \frac{1}{7/2} = \frac{2}{7} = \text{Rational number}$$

The LCM of $$N_1, N_2, N_3 = \frac{7}{2} \times 3 = \frac{21}{2}$$

$\therefore$ The given signal $x(n)$ is periodic with fundamental period $N = 10.5$.

3. Causal and Non-causal Signals

A discrete-time signal $x(n)$ is said to be causal if $x(n) = 0$ for $n < 0$, otherwise the signal is non-causal. A discrete-time signal $x(n)$ is said to be anti-causal if $x(n) = 0$ for $n > 0$.

A causal signal does not exist for negative time and an anti-causal signal does not exist for positive time. A signal which exists in positive as well as negative time is called a non-casual signal.

$u(n)$ is a causal signal and $u(-n)$ an anti-causal signal, whereas $x(n) = 1$ for $-2 \le n \le 3$ is a non-causal signal.

Example 4 Find which of the following signals are causal or non-causal.

(a) $x(n) = u(n+4) - u(n-2)$

(b) $x(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4)$

(c) $x(n) = u(-n)$

Solution:

(a) Given $x(n) = u(n+4) - u(n-2)$
The given signal exists from $n = -4$ to $n = 1$. Since $x(n) \neq 0$ for $n < 0$, it is non-causal.

(b) Given $x(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4)$
The given signal exists for $n < 0$ also. So it is non-causal.

(c) Given $x(n) = u(-n)$
The given signal exists only for $n < 0$. So it is anti-causal. It can be called non-causal also.

4. Energy and Power Signals

Signals may also be classified as energy signals and power signals. However there are some signals which can neither be classified as energy signals nor power signals.

The total energy E of a discrete-time signal $x(n)$ is defined as:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

and the average power P of a discrete-time signal $x(n)$ is defined as:

$$P = \underset{N\to\infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

or $P = \dfrac{1}{N}\displaystyle\sum_{n=0}^{n-1} |x(n)|^2$ for a digital signal with $x(n) = 0$ for $n < 0$.

A signal is said to be an energy signal if and only if its total energy E over the interval $(-\infty, \infty)$ is finite (i.e., $0 < E < \infty$). For an energy signal, average power $P = 0$. Non-periodic signals which are defined over a finite time (also called time limited signals) are the examples of energy signals. Since the energy of a periodic signal is always either zero or infinite, any periodic signal cannot be an energy signal.

A signal is said to be a power signal, if its average power P is finite (i.e., $0 < P < \infty$). For a power signal, total energy $E = \infty$. Periodic signals are the examples of power signals. Every bounded and periodic signal is a power signal. But it is true that a power signal is not necessarily a bounded and periodic signal.

Both energy and power signals are mutually exclusive, i.e. no signal can be both energy signal and power signal.

The signals that do not satisfy the above properties are neither energy signals nor power signals. For example, $x(n) = u(n)$, $x(n) = nu(n)$, $x(n) = n^2u(n)$.

These are signals for which neither P nor E are finite. If the signals contain infinite energy and zero power or infinite energy and infinite power, they are neither energy nor power signals.

If the signal amplitude becomes zero as $|n| \to \infty$, it is an energy signal, and if the signal amplitude does not become zero as $|n| \to \infty$, it is a power signal.

Example 5 Find which of the following signals are energy signals, power signals, neither energy nor power signals:

(a) $\left(\frac{1}{2}\right)^n u(n)$ (b) $e^{j[(\pi/3)n+(\pi/2)]}$

(c) $\sin\left(\frac{\pi}{3}n\right)$ (d) $u(n) - u(n-6)$

(e) $nu(n)$ (f) $r(n) - r(n-4)$

Solution:

(a) Given $$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

Energy of the signal $E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} |x(n)|^2$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \left[\left(\frac{1}{2}\right)^n\right]^2 u(n)$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-(1/4)} = \frac{4}{3} \text{ joule}$$

Power of the signal $P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \left[\frac{1-(1/4)^{N+1}}{1-(1/4)}\right]$$

$$= 0$$

The energy is finite and power is zero. Therefore, $x(n)$ is an energy signal.

(b) Given $$x(n) = e^{j[(\pi/3)n + (\pi/2)]}$$

Energy of the signal $E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \left|e^{j[(\pi/3)n + (\pi/2)]}\right|^2$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} 1$$

$$= \underset{N\to\infty}{\text{Lt}} [2N+1] = \infty$$

Power of the signal $P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|e^{j[(\pi/3)n + (\pi/2)]}\right|^2$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} [2N+1] = 1 \text{ watt}$$

The energy is infinite and the power is finite. Therefore, it is a power signal.

(c) Given $$x(n) = \sin\left(\frac{\pi}{3}n\right)$$

Energy of the signal $$E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{3}n\right)$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \frac{1-\cos[(2\pi/3)\,n]}{2}$$

$$= \frac{1}{2} \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \left(1-\cos\frac{2\pi}{3}n\right)$$

$$= \infty$$

Power of the signal $$P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{3}n\right)$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos[(2\pi/3)\,n]}{2}$$

$$= \frac{1}{2} \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1}[2N+1] = \frac{1}{2} \text{ watt}$$

The energy is infinite and power is finite. Therefore, it is a power signal.

(d) Given $$x(n) = u(n) - u(n-6)$$

Energy of the signal $$E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} [u(n)-u(n-6)]^2$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=0}^{5} 1 = 6 \text{ joule}$$

Power of the signal $$P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} [u(n)-u(n-6)]^2$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=0}^{5} 1 = 0$$

Energy is finite and power is zero. Therefore, it is an energy signal.

(e) Given $$x(n) = nu(n)$$

Energy of the signal $$E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} [n]^2\, u(n)$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=0}^{N} [n^2]$$

$$= \infty$$

$$\text{Power of the signal } P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} [n]^2 u(n)$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

$$= \infty$$

Energy is infinite and power is also infinite. Therefore, it is neither energy signal nor power signal.

Example 6 Find whether the signal

$$x(n) = \begin{cases} n^2 & 0 \le n \le 3 \\ 10 - n & 4 \le n \le 6 \\ n & 7 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

is a power signal or an energy signal. Also find the energy and power of the signal.

Solution: The given signal is a non-periodic finite duration signal. So it has finite energy and zero average power. So it is an energy signal.

$$\text{Energy of the signal } E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} (n)^2$$

$$= \sum_{n=0}^{3} n^4 + \sum_{n=4}^{6} (100 + n^2 - 20n) + \sum_{n=7}^{9} n^2$$

$$= (0 + 1 + 16 + 81) + (36 + 25 + 16) + (49 + 64 + 81)$$

$$= 369 < \infty \text{ joule}$$

$$\text{Power of the signal } P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \left[\sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} (n)^2 \right]$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} [369] = 0$$

Here energy is finite and power is zero. So it is an energy signal.

5. Even and Odd Signals

Any signal $x(n)$ can be expressed as sum of even and odd components. That is

$$x(n) = x_e(n) + x_o(n)$$

where $x_e(n)$ is even components and $x_o(n)$ is odd components of the signal.

Even (symmetric) signal

A discrete-time signal $x(n)$ is said to be an even (symmetric) signal if it satisfies the condition:

$$x(n) = x(-n) \quad \text{for all } n$$

Even signals are symmetrical about the vertical axis or time origin. Hence they are also called symmetric signals: cosine sequence is an example of an even signal. Some even signals are shown in Figure 1. (a). An even signal is identical to its reflection about the origin. For an even signal $x_0(n) = 0$.

Odd (anti-symmetric) signal

A discrete-time signal $x(n)$ is said to be an odd (anti-symmetric) signal if it satisfies the condition:

$$x(-n) = -x(n) \quad \text{for all } n$$

Odd signals are anti-symmetrical about the vertical axis. Hence they are called anti-symmetric signals. Sinusoidal sequence is an example of an odd signal. For an odd signal $x_e(n) = 0$. Some odd signals are shown in Figure 1. (b).

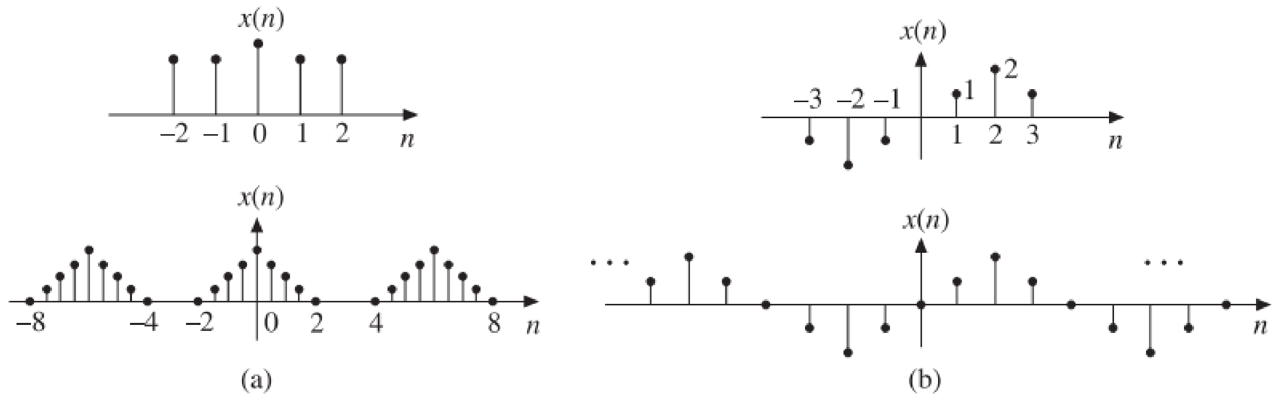


Figure 1 (a) Even sequences (b) Odd sequences.

Evaluation of even and odd parts of a signal

We have $$x(n) = x_e(n) + x_o(n)$$

$\therefore$ $$x(-n) = x_e(-n) + x_o(-n) = x_e(n) - x_o(n)$$

$$x(n) + x(-n) = x_e(n) + x_o(n) + x_e(n) - x_o(n) = 2x_e(n)$$

$\therefore$ $$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x(n) - x(-n) = [x_e(n) + x_o(n)] - [x_e(n) - x_o(n)] = 2x_o(n)$$

$\therefore$ $$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

The product of two even or odd signals is an even signal and the product of even signal and odd signal is an odd signal.

We can prove this as follows:

Let $$x(n) = x_1(n)\, x_2(n)$$

(a) If $x_1(n)$ and $x_2(n)$ are both even, i.e.

$$x_1(-n) = x_1(n)$$

and $$x_2(-n) = x_2(n)$$

Then $$x(-n) = x_1(-n)x_2(-n) = x_1(n)x_2(n) = x(n)$$

Therefore, $x(n)$ is an even signal.

If $x_1(n)$ and $x_2(n)$ are both odd, i.e.

$$x_1(-n) = -x_1(n)$$

and $$x_2(-n) = -x_2(n)$$

Then $$x(-n) = x_1(-n)x_2(-n) = [-x_1(n)][-x_2(n)] = x_1(n)x_2(n) = x(n)$$

Therefore, $x(n)$ is an even signal.

(b) If $x_1(n)$ is even and $x_2(n)$ is odd, i.e.

$$x_1(-n) = x_1(n)$$

and $$x_2(-n) = -x_2(n)$$

Then $$x(-n) = x_1(-n)x_2(-n) = -x_1(n)x_2(n) = -x(n)$$

Therefore, $x(n)$ is an odd signal.

Thus, the product of two even signals or of two odd signals is an even signal, and the product of even and odd signals is an odd signal.

Every signal need not be either purely even signal or purely odd signal, but every signal can be decomposed into sum of even and odd parts.

Example 7 Find the even and odd components of the following signals:

(a) $x(n) = \left\{-3, 1, \underset{\uparrow}{2}, -4, 2\right\}$ (b) $x(n) = \left\{-2, 5, \underset{\uparrow}{1}, -3\right\}$

(c) $x(n) = \left\{\underset{\uparrow}{5}, 4, 3, 2, 1\right\}$ (d) $x(n) = \left\{5, 4, 3, 2, \underset{\uparrow}{1}\right\}$

Solution:

(a) Given $$x(n) = \left\{-3, 1, \underset{\uparrow}{2}, -4, 2\right\}$$

$$\therefore \quad x(-n) = \left\{2, -4, \underset{\uparrow}{2}, 1, -3\right\}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}\left[-3+2, 1-4, 2+2, -4+1, 2-3\right]$$

$$= \left\{-0.5, -1.5, \underset{\uparrow}{2}, -1.5, -0.5\right\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}\left[-3-2, 1+4, 2-2, -4-1, 2+3\right]$$

$$= \left\{-2.5, 2.5, \underset{\uparrow}{0}, -2.5, 2.5\right\}$$

(b) Given $x(n) = \left\{-2, 5, \underset{\uparrow}{1}, -3\right\}$

$$x(-n) = \left\{-3, \underset{\uparrow}{1}, 5, -2\right\}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}\left[-2+0, 5-3, 1+1, -3+5, 0-2\right]$$

$$= \left\{-1, 1, \underset{\uparrow}{1}, 1, -1\right\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}\left[-2-0, 5+3, 1-1, -3-5, 0+2\right]$$

$$= \left\{-1, 4, \underset{\uparrow}{0}, -4, 1\right\}$$

(c) Given $x(n) = \left\{\underset{\uparrow}{5}, 4, 3, 2, 1\right\}$

$$n = 0, 1, 2, 3, 4$$

$\therefore$ $x(n) = \underset{\uparrow}{5}, 4, 3, 2, 1$

$$x(-n) = 1, 2, 3, 4, \underset{\uparrow}{5}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}[1, 2, 3, 4, 5 \underset{\uparrow}{+} 5, 4, 3, 2, 1]$$

$$= \left\{0.5, 1, 1.5, 2, \underset{\uparrow}{5}, 2, 1.5, 1, 0.5\right\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[-1, -2, -3, -4, 5 \underset{\uparrow}{-} 5, 4, 3, 2, 1]$$

$$= \left\{-0.5, -1, -1.5, -2, \underset{\uparrow}{0}, 2, 1.5, 1, 0.5\right\}$$

(d) Given $x(n) = \left\{5, 4, 3, 2, \underset{\uparrow}{1}\right\}$

$$n = -4, -3, -2, -1, 0$$

$\therefore$ $x(n) = 5, 4, 3, 2, \underset{\uparrow}{1}$

$$x(-n) = \underset{\uparrow}{1}, 2, 3, 4, 5$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}[5, 4, 3, 2, 1 \underset{\uparrow}{+} 1, 2, 3, 4, 5]$$

$$= [2.5, 2, 1.5, 1, \underset{\uparrow}{1}, 1, 1.5, 2, 2.5]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[5, 4, 3, 2, 1 \underset{\uparrow}{-} 1, -2, -3, -4, -5]$$

$$= \frac{1}{2}[2.5, 2, 1.5, 1, \underset{\uparrow}{0}, -1, -1.5, -2, -2.5]$$

When the signal is given as a waveform

The even part of the signal can be found by folding the signal about the y-axis and adding the folded signal to the original signal and dividing the sum by two. The odd part of the signal can be found by folding the signal about y-axis and subtracting the folded signal from the original signal and dividing the difference by two as illustrated in Figure 2.

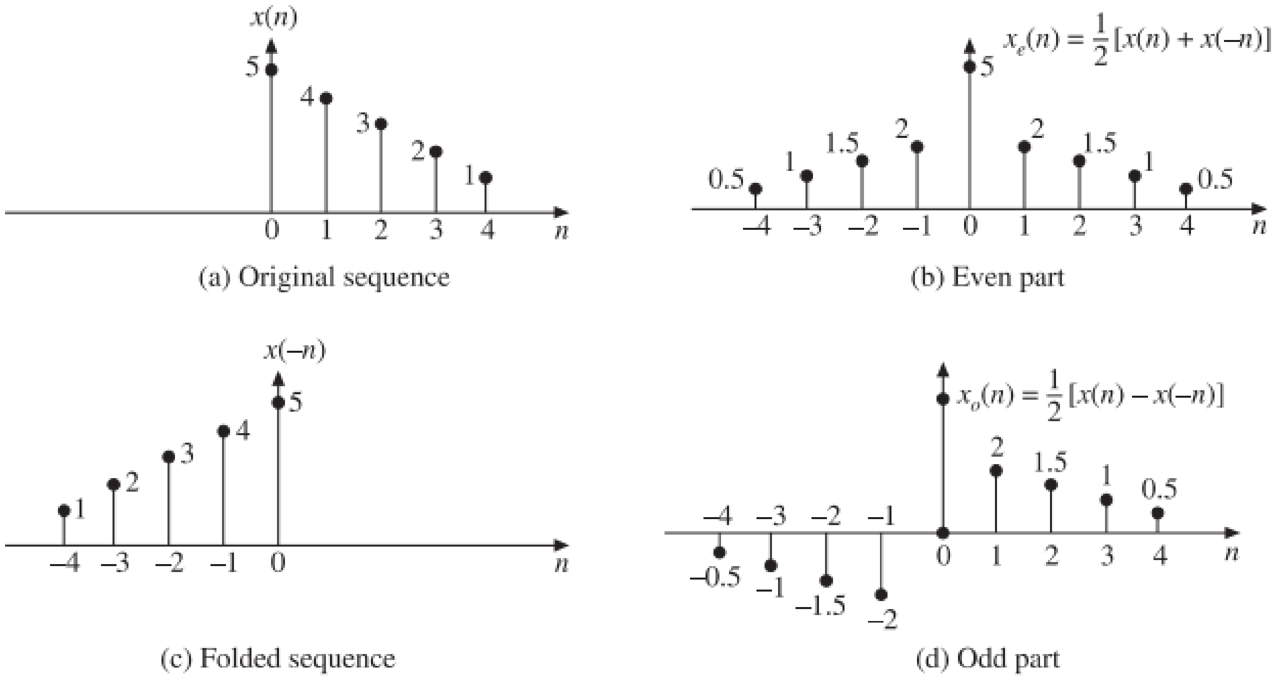


Figure 2 Graphical evaluation of even and odd parts.