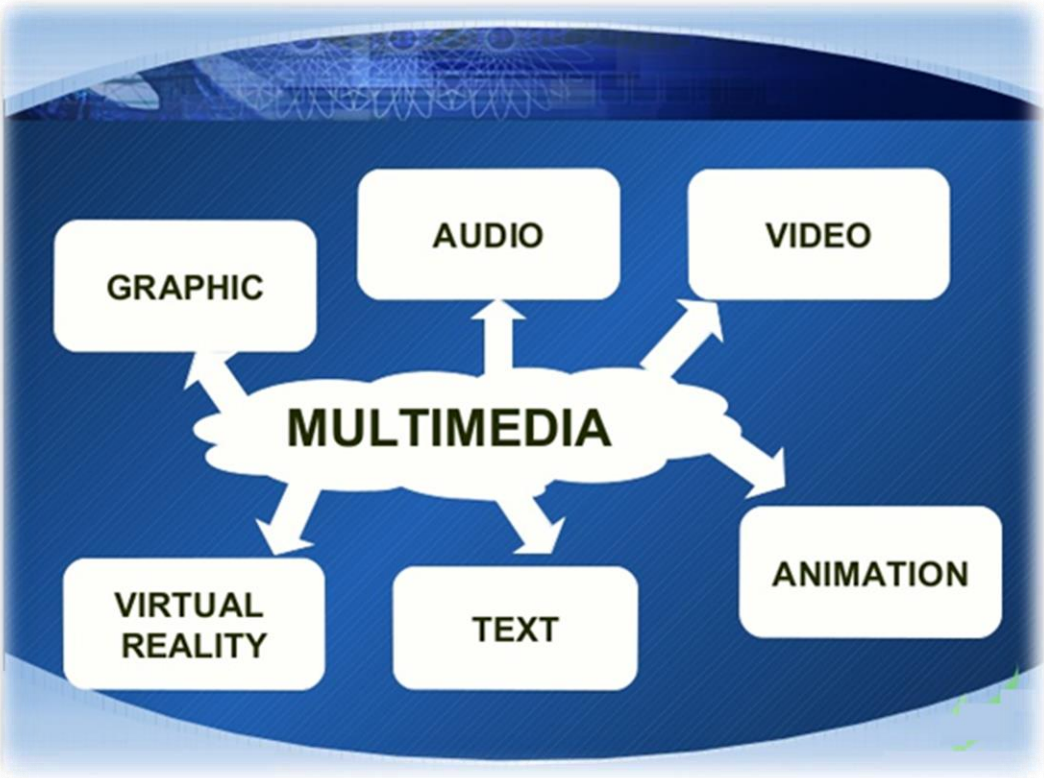

Tikrit University

Computer Science Dept.

Master Degree

Lecture 6

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• Discrete Time Signal Representation

Signal representation There are several ways to represent a discrete-time signal. The more widely used representations are illustrated in Table 1 by means of a simple example. Figure 1 also shows a pictorial representation of a sampled signal using index n as well as sampling instances $t = nT$. We will use one of the two representations as appropriate in a given situation.

The *duration* or *length* L_x of a discrete-time signal $x[n]$ is the number of samples from the first nonzero sample $x[n_1]$ to the last nonzero sample $x[n_2]$, that is $L_x = n_2 - n_1 + 1$. The range $n_1 \le n \le n_2$ is denoted by $[n_1, n_2]$ and it is called the *support* of the sequence.

Table 1 Discrete-time signal representations.

Representation	Example
Functional	$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \ge 0 \\ 0, & n < 0 \end{cases}$
Tabular	$\begin{array}{c\|cccccccc} n & \ldots & -2 & -1 & 0 & 1 & 2 & 3 & \ldots \\ \hline x[n] & \ldots & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \ldots \end{array}$
Sequence	$x[n] = \{ \ldots\ 0\ \underset{\uparrow}{1}\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\ \ldots \}$
Pictorial	

[1] The symbol ↑ denotes the index $n = 0$; it is omitted when the table starts at $n = 0$.

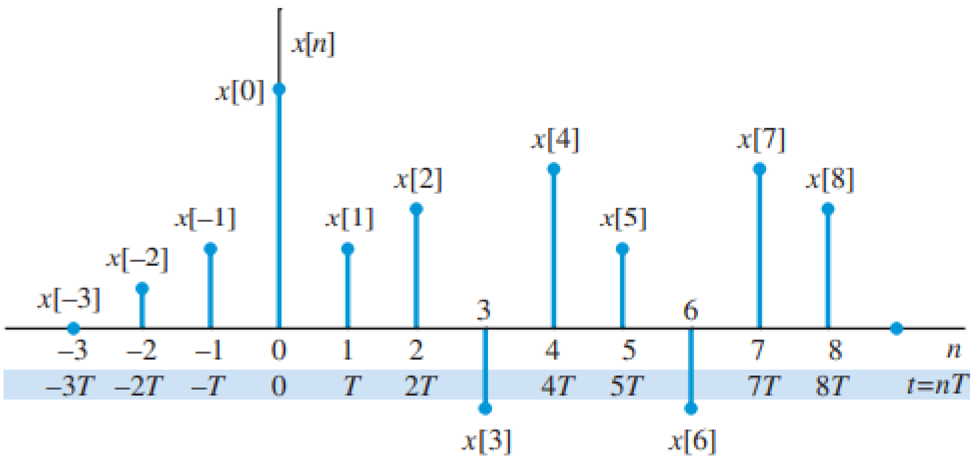


Figure 1 Representation of a Sampled Signal.

- **Basic Operations on Signals**

When we process a sequence, this sequence may undergo several manipulations involving the independent variable or the amplitude of the signal. The basic operations on sequences are as follows:

1. Time shifting
2. Time reversal
3. Time scaling
4. Amplitude scaling
5. Signal addition
6. Signal multiplication

The first three operations correspond to transformation in independent variable n of a signal. The last three operations correspond to transformation on amplitude of a signal.

1. Time Shifting

The time shifting of a signal may result in time delay or time advance. The time shifting operation of a discrete- time signal $x(n)$ can be represented by $y(n) = x(n - k)$. This shows that the signal $y(n)$ can be obtained by time shifting the signal $x(n)$ by k units. If k is positive, it is delay and the shift is to the right, and if k is negative, it is advance and the shift is to the left. An arbitrary signal $x(n)$ is shown in Figure 2(a). $x(n - 3)$ which is obtained by shifting $x(n)$ to the right by 3 units (i.e. delay $x(n)$ by 3 units) is shown in Figure 2(b). $x(n + 2)$ which is obtained by shifting $x(n)$ to the left by 2 units (i.e. advancing $x(n)$ by 2 units) is shown in Figure 2(c).

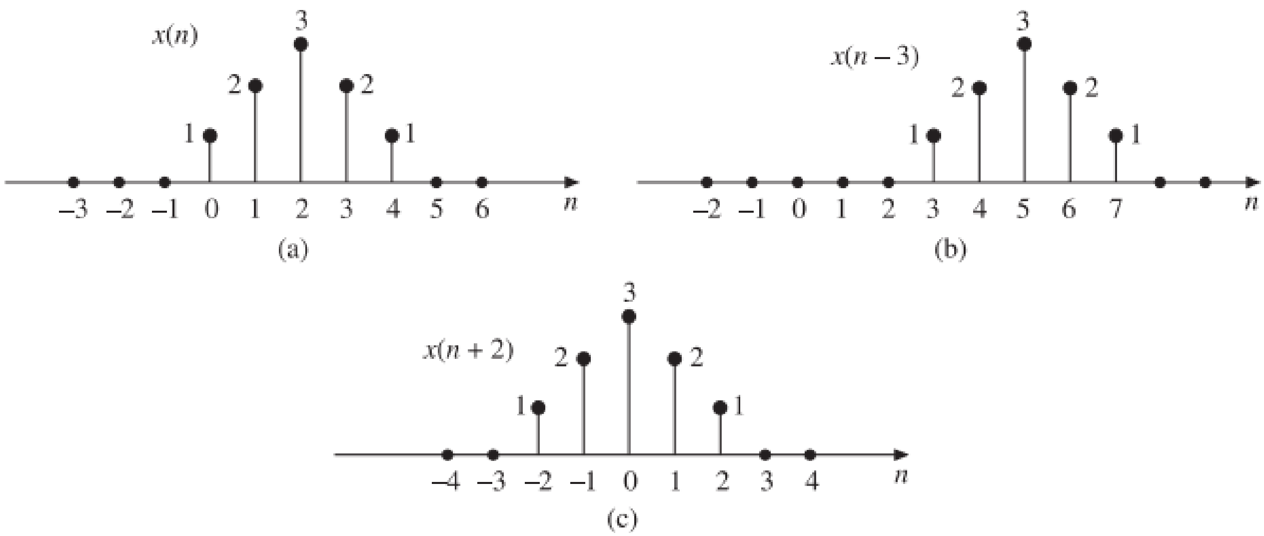


Figure 2 (a) Sequence x(n) (b) x(n – 3) (c) x(n + 2).

2. Time Reversal

The time reversal also called time folding of a discrete-time signal $x(n)$ can be obtained by folding the sequence about $n = 0$. The time reversed signal is the reflection of the original signal. It is obtained by replacing the independent variable n by $-n$. Figure 3(a) shows an arbitrary discrete-time signal $x(n)$, and its time reversed version $x(-n)$ is shown in Figure 3(b). Figure 3 [(c) and (d)] shows the delayed and advanced versions of reversed signal $x(-n)$. The signal $x(-n + 3)$ is obtained by delaying (shifting to the right) the time reversed signal $x(-n)$ by 3 units of time. The signal $x(-n - 3)$ is obtained by advancing (shifting to the left) the time reversed signal $x(-n)$ by 3 units of time. Figure 4 shows other examples for time reversal of signals.

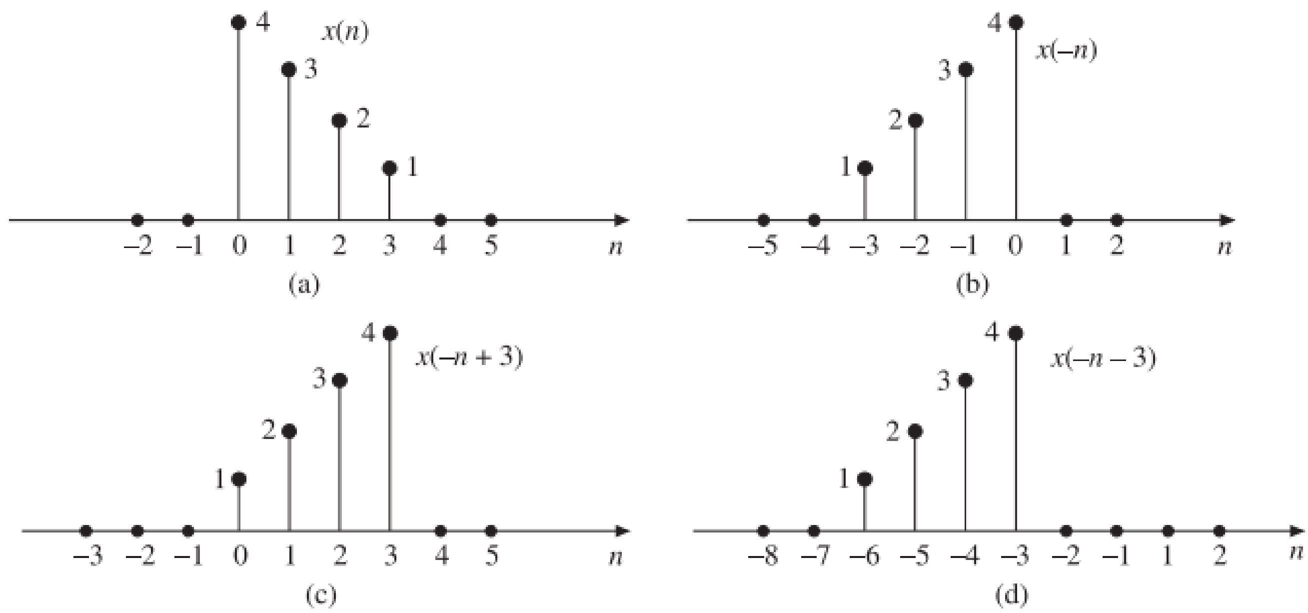


Figure 3 (a) Original signal $x(n)$ (b) Time reversed signal $x(-n)$ (c) Time reversed and delayed signal $x(-n+3)$ (d) Time reversed and advanced signal $x(-n-3)$.

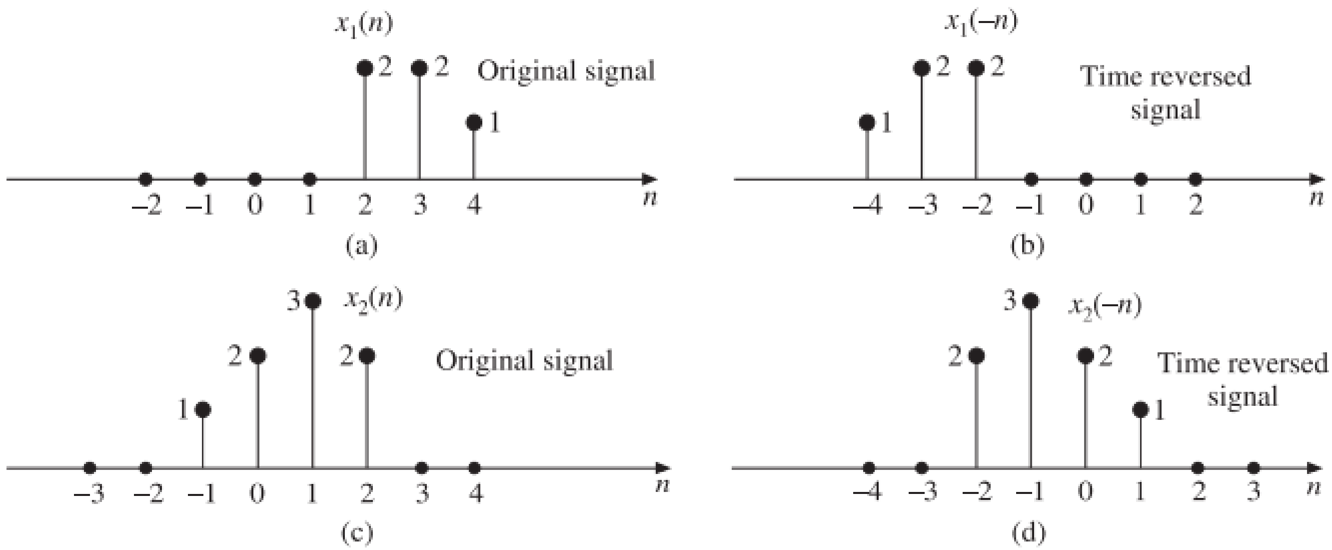


Figure 4 Time reversal operations.

EXAMPLE Sketch the following signals:

(a) $u(n+2)\,u(-n+3)$ (b) $x(n) = u(n+4) - u(n-2)$

Solution:

(a) Given $x(n) = u(n+2)\,u(-n+3)$

The signal $u(n+2)\,u(-n+3)$ can be obtained by first drawing the signal $u(n+2)$ as shown in **Figure 5** (a), then drawing $u(-n+3)$ as shown in **Figure 5** (b),

and then multiplying these sequences element by element to obtain $u(n+2)\,u(-n+3)$ as shown in **Figure 5** (c).

$x(n) = 0$ for $n < -2$ and $n > 3$; $x(n) = 1$ for $-2 < n < 3$

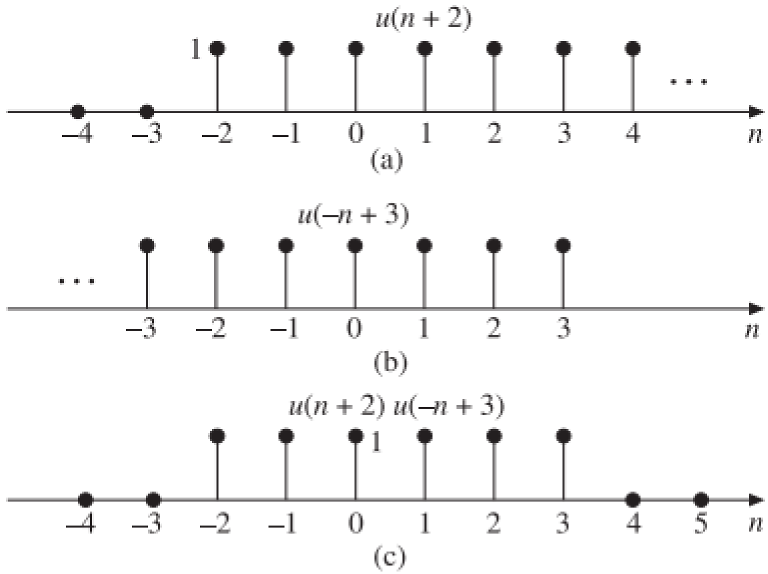


Figure 5 Plots of (a) $u(n+2)$ (b) $u(-n+3)$ (c) $u(n+2)\,u(-n+3)$.

(b) Given $x(n) = u(n+4) - u(n-2)$

The signal $u(n+4) - u(n-2)$ can be obtained by first plotting $u(n+4)$ as shown in **Figure 6** (a), then plotting $u(n-2)$ as shown in **Figure 6** (b), and then subtracting each element of $u(n-2)$ from the corresponding element of $u(n+4)$ to obtain the result shown in **Figure 6** (c).

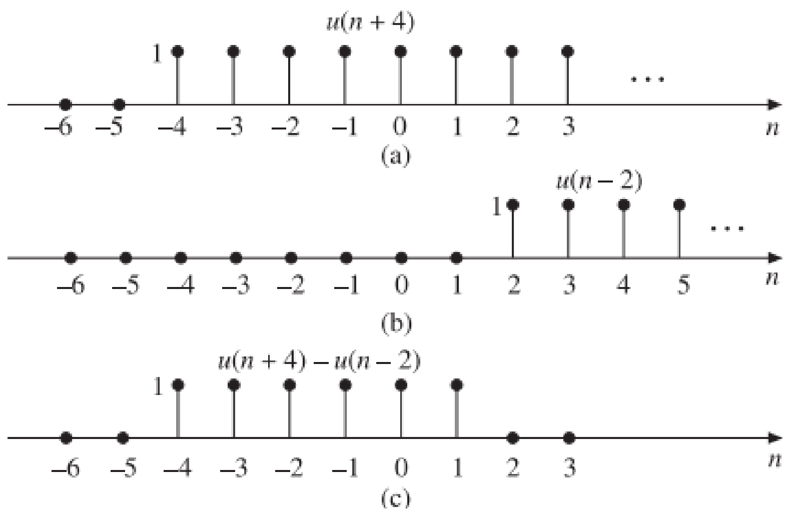


Figure 6 Plots of (a) $u(n+4)$ (b) $u(n-2)$ (c) $u(n+4) - u(n-2)$.

3. Amplitude Scaling

The amplitude scaling of a discrete-time signal can be represented by

$$y(n) = ax(n)$$

where a is a constant.

The amplitude of $y(n)$ at any instant is equal to a times the amplitude of $x(n)$ at that instant. If $a > 1$, it is amplification and if $a < 1$, it is attenuation. Hence the amplitude is rescaled. Hence the name amplitude scaling.

Figure 7 (a) shows a signal $x(n)$ and **Figure** 7 (b) shows a scaled signal $y(n) = 2x(n)$.

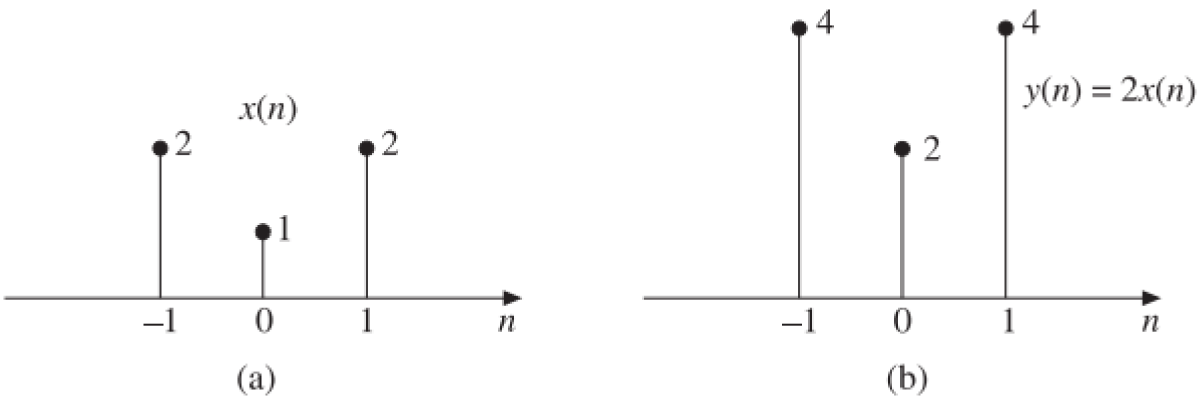


Figure 7 Plots of (a) Signal $x(n)$ (b) $y(n) = 2x(n)$.

4. Time Scaling

Time scaling may be time expansion or time compression. The time scaling of a discrete-time signal $x(n)$ can be accomplished by replacing n by an in it. Mathematically, it can be expressed as:

$$y(n) = x(an)$$

When $a > 1$, it is time compression and when $a < 1$, it is time expansion.

Let $x(n)$ be a sequence as shown in **Figure 8** (a). If $a = 2$, $y(n) = x(2n)$. Then

$$y(0) = x(0) = 1$$
$$y(-1) = x(-2) = 3$$
$$y(-2) = x(-4) = 0$$
$$y(1) = x(2) = 3$$
$$y(2) = x(4) = 0$$

and so on.

So to plot $x(2n)$ we have to skip odd numbered samples in $x(n)$.

We can plot the time scaled signal $y(n) = x(2n)$ as shown in **Figure 8** (b). Here the signal is compressed by 2.

If $a = (1/2)$, $y(n) = x(n/2)$, then

$$y(0) = x(0) = 1$$
$$y(2) = x(1) = 2$$
$$y(4) = x(2) = 3$$
$$y(6) = x(3) = 4$$
$$y(8) = x(4) = 0$$
$$y(-2) = x(-1) = 2$$
$$y(-4) = x(-2) = 3$$
$$y(-6) = x(-3) = 4$$
$$y(-8) = x(-4) = 0$$

We can plot $y(n) = x(n/2)$ as shown in **Figure 8** (c). Here the signal is expanded by 2. All odd components in $x(n/2)$ are zero because $x(n)$ does not have any value in between the sampling instants.

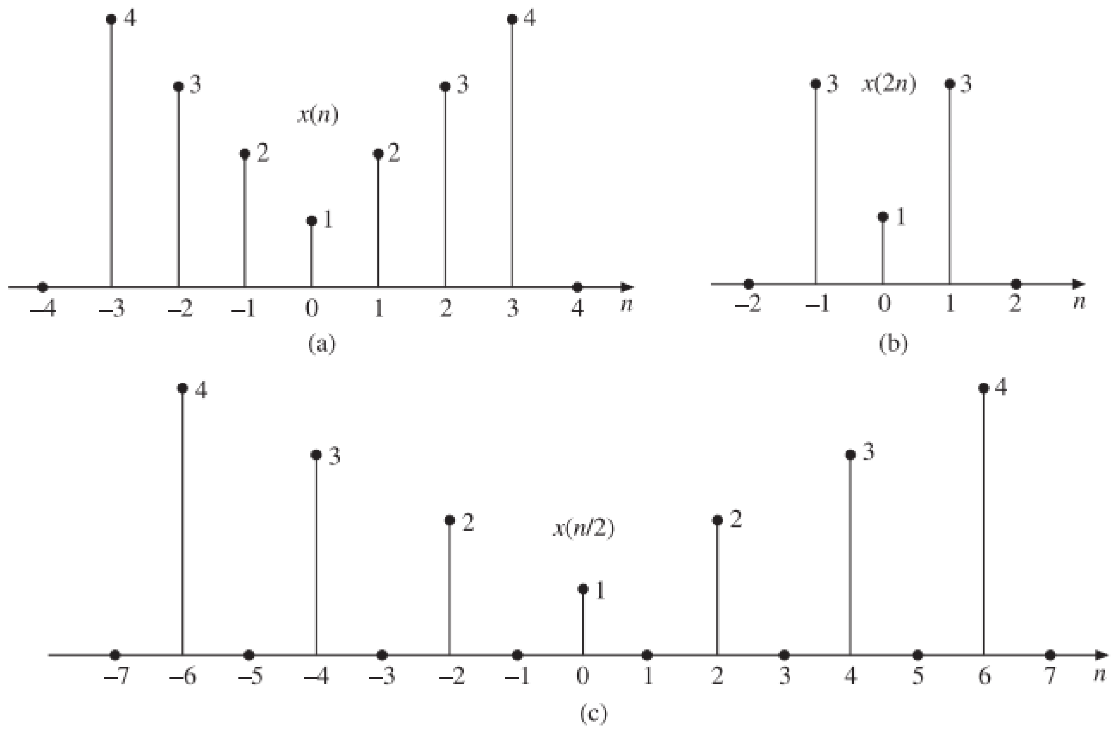


Figure 8 Discrete-time scaling (a) Plot of $x(n)$ (b) Plot of $x(2n)$ (c) Plot of $x(n/2)$.

Time scaling is very useful when data is to be fed at some rate and is to be taken out at a different rate.

5. Signal Addition

In discrete-time domain, the sum of two signals $x_1(n)$ and $x_2(n)$ can be obtained by adding the corresponding sample values and the subtraction of $x_2(n)$ from $x_1(n)$ can be obtained by subtracting each sample of $x_2(n)$ from the corresponding sample of $x_1(n)$ as illustrated below.

If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$

Then $x_1(n) + x_2(n) = \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\} = \{3, 5, 7, 2, 3\}$

and $x_1(n) - x_2(n) = \{1 - 2, 2 - 3, 3 - 4, 1 - 1, 5 + 2\} = \{-1, -1, -1, 0, 7\}$

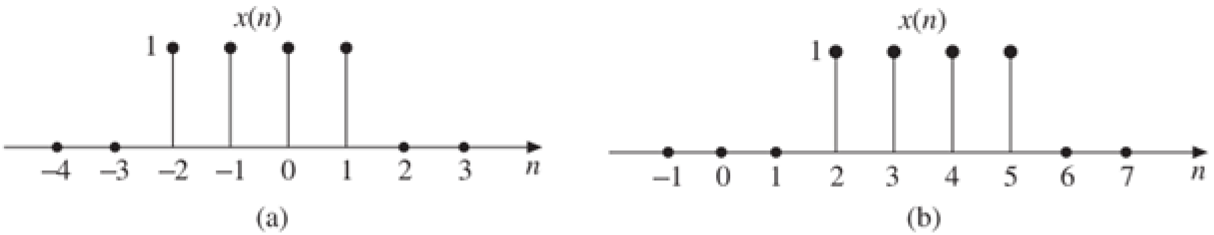
6. Signal Multiplication

The multiplication of two discrete-time sequences can be performed by multiplying their values at the sampling instants as shown below.

If $x_1(n) = \{1, -3, 2, 4, 1.5\}$ and $x_2(n) = \{2, -1, 3, 1.5, 2\}$

Then
$$x_1(n)\, x_2(n) = \{1\times 2, -3\times -1, 2\times 3, 4\times 1.5, 1.5\times 2\}$$
$$= \{2, 3, 6, 6, 3\}$$

Example 1 Express the signals shown in **Figure 9** as the sum of singular functions.



Solution:

(a) The given signal shown in **Figure 9** (a) is:

$$x(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$x(n) = \begin{cases} 0 & \text{for } n \le -3 \\ 1 & \text{for } -2 \le n \le 1 \\ 0 & \text{for } n \ge 2 \end{cases}$$

$$\therefore \quad x(n) = u(n+2) - u(n-2)$$

(b) The signal shown in **Figure 9** (b) is:

$$x(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$x(n) = \begin{cases} 0 & \text{for } n \le 1 \\ 1 & \text{for } 2 \le n \le 5 \\ 0 & \text{for } n \ge 6 \end{cases}$$

$$\therefore \quad x(n) = u(n-2) - u(n-6)$$

Example 2

Consider the length – 7 sequences defined for $-3 \le n \le 3$:

$$x(n) = \{3, -2, 0, 1, 4, 5, 2\}$$

$$y(n) = \{0, 7, 1, -3, 4, 9, -2\}$$

$$w(n) = \{-5, 4, 3, 6, -5, 0, 1\}$$

Generate the following sequence:

(a) $u(n) = x(n) + y(n)$
(b) $v(n) = x(n) \cdot w(n)$
(c) $s(n) = y(n) - w(n)$
(d) $r(n) = 4.5y(n)$

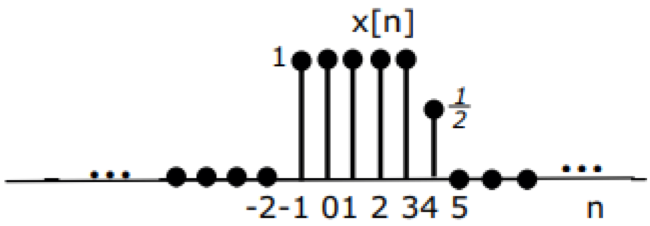
Solution

(a) $u(n) = x(n) + y(n) = \{3, 5, 1, -2, 8, 14, 0\}$
(b) $v(n) = x(n) \cdot w(n) = \{-15, -8, 0, 6, -20, 0, 2\}$
(c) $s(n) = y(n) - w(n) = \{5, 3, -2, -9, 9, 9, -3\}$
(d) $r(n) = 4.5y(n) = \{0, 31.5, 4.5, -13.5, 19, 40.5, -9\}$.

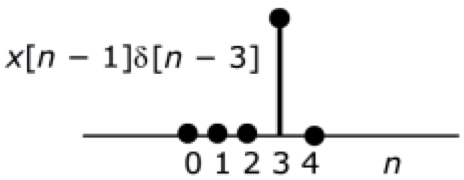
Example 3

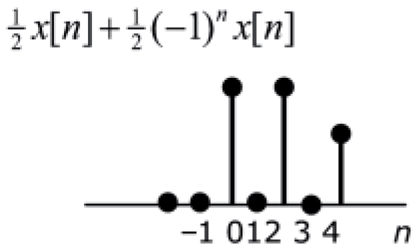
A **DT** signal $x[n]$ is shown in Figure. Sketch and label carefully each of the following signals.

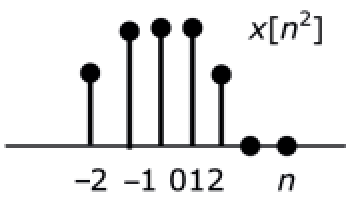
(i) $x[n-1]\delta[n-3]$
(ii) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$
(iii) $x[n^2]$



Solution

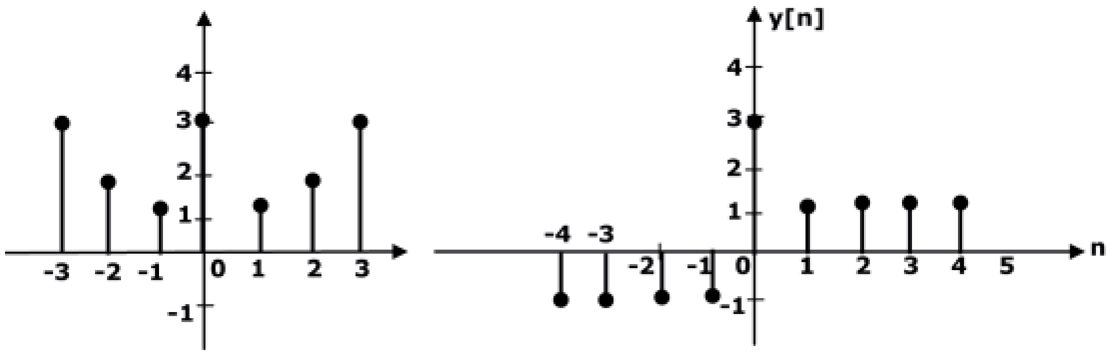






Example 4

Let $x[n]$ and $y[n]$ be given in Figures, respectively.



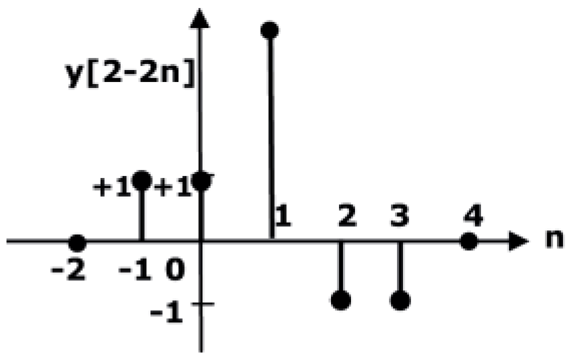
Carefully sketch the following signals.

(a) $y[2-2n]$
(b) $x[n-2]+y[n+2]$
(c) $x[2n]+y[n-4]$
(d) $x[n+2]y[n-2]$

Solution

(a) $y[2-2n]$

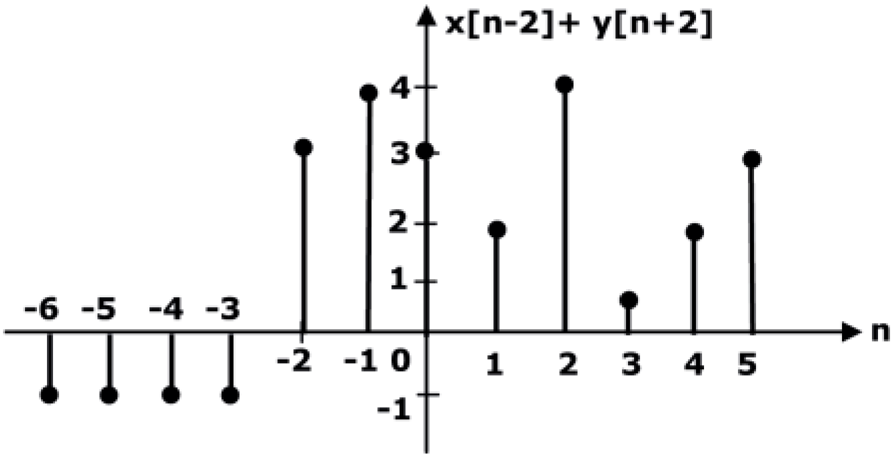
$$y[2-2n] = \begin{cases} 1, & n = 0, -1 \\ -1, & n = 2, 3 \\ 3, & n = 1 \end{cases}$$



(b) $x[n-2]+y[n+2]$

$$x[n-2] = \begin{cases} 1, & n = 1,\ 3 \\ 2, & n = 0,\ 4 \\ 3, & n = -1, 2, 5 \\ 0, & n = \text{rest} \end{cases}$$

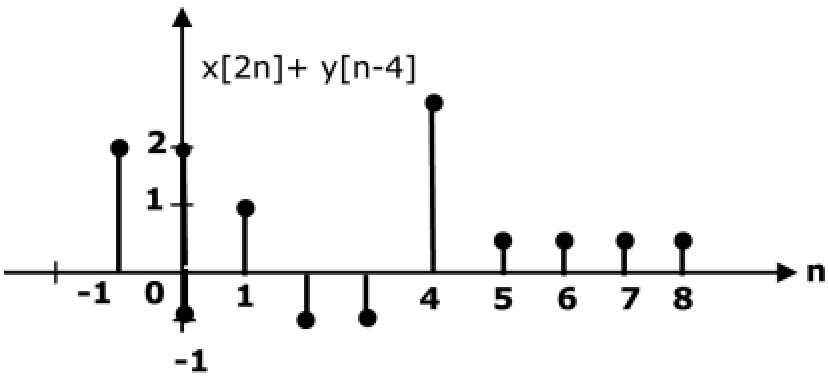
$$y[n+2] = \begin{cases} 1, & n = -1, 0, 1, 2 \\ -1, & n = -3, -4, -5, -6 \\ 3, & n = -2 \end{cases}$$



(c) $x[2n] + y[n-4]$

$$x[2n] = \begin{cases} 2, & n = \pm 1 \\ 0, & n = 3 \end{cases}$$

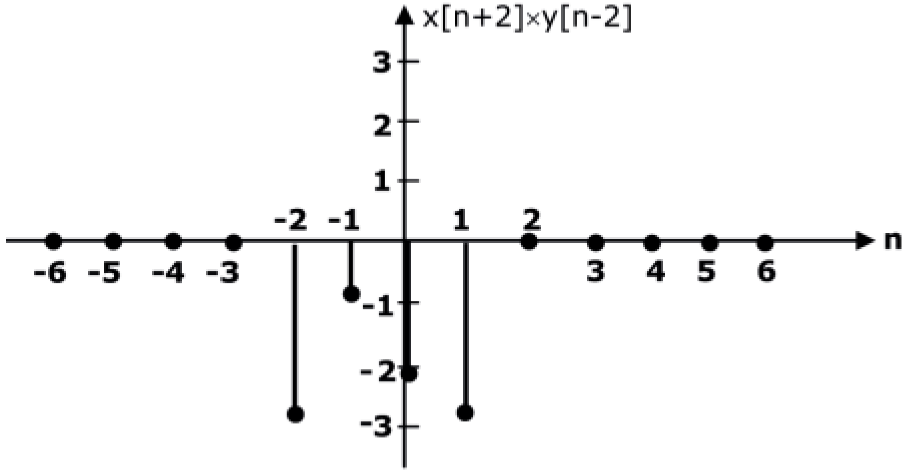
$$y[n-4] = \begin{cases} 1, & n = 5, 6, 7, 8 \\ -1, & n = 0, 1, 2, 3 \\ 3, & n = 4 \end{cases}$$



(d) $x[n+2]y[n-2]$

$$x[n+2] = \begin{cases} 1, & n = -3,\ -1 \\ 2, & n = -4, 0 \\ 3, & n = -5,\ -2, 1 \end{cases}$$

$$y[n-2] = \begin{cases} 1, & n = 3, 4, 5, 6 \\ -1, & n = 1, 0, -1, -2 \\ 3, & n = \ 2 \end{cases}$$


x[n+2]×y[n-2]
3
2
1
-1
-2
-3
n
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6