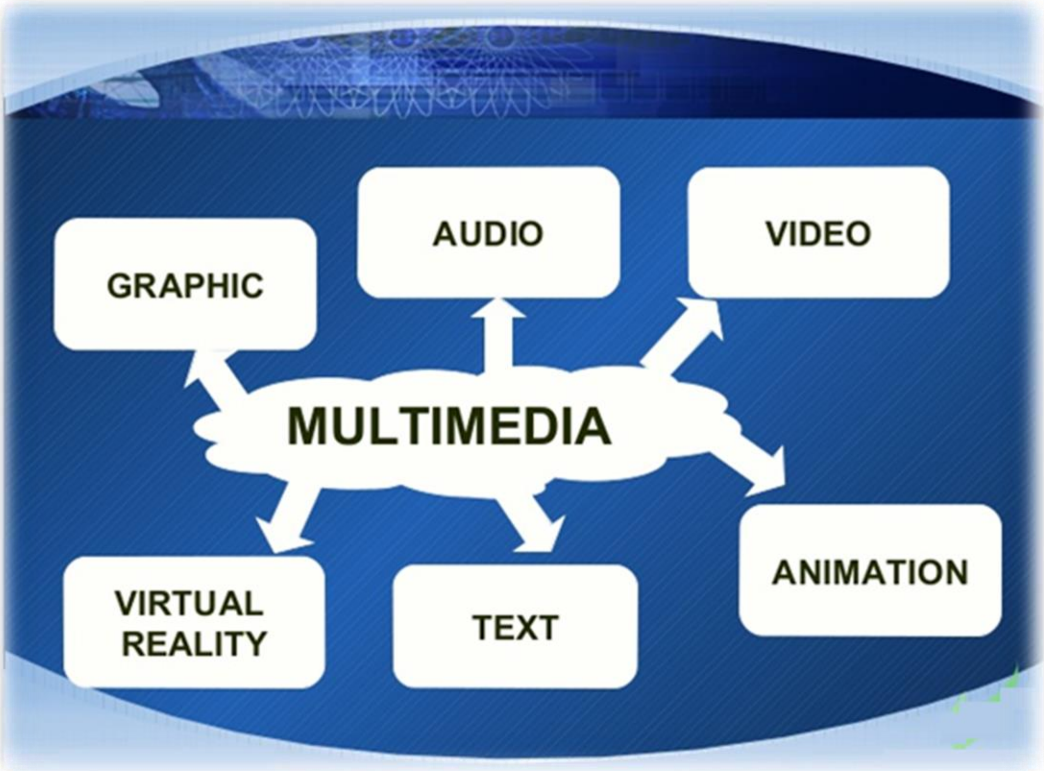


# Tikrit University

## Computer Science Dept.

## Master Degree

## Lecture 5

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- **Signals** are time - varying quantities which carry information in their patterns of variation. They may be, for example, audio signals (speech, music), images or video signals, sonar signals or ultrasound, biological signals such as the electrical pulses from the heart, communications signals, or many other types. The manipulation of this information involves the acquisition, storage, transmission, and transformation of signals. The speech signal, shown as a time waveform in Figure 1, represents the variations of acoustic pressure converted into an electric signal by a microphone. A signal, which is continuous in nature is known as continuous signal.

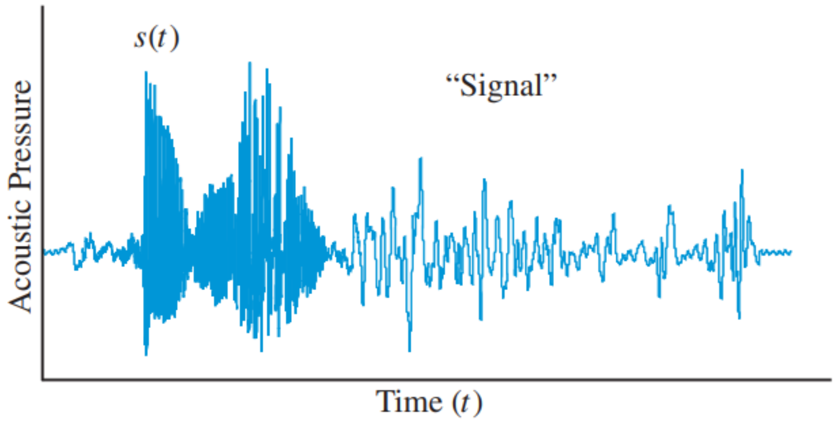


**Figure 1:** Example of a recording of speech signal

General format of a sinusoidal signal is shown in Figure 2 and it is:

$$v(t) = A\sin(\omega t + \Phi)$$

where

$A$ is the magnitude of the signal $v(t)$
$\omega$ is the angular frequency ($\omega = 2\pi f$)
$f$ is the frequency in hertz ($f = 1/T$)
$\Phi$ is the phase angle in radians

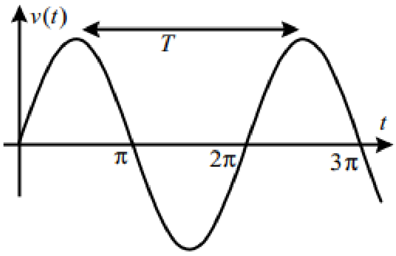


**Figure 2:** a sinusoidal signal

- To simplify the analysis and design of signal processing systems it is almost always necessary to represent signals by mathematical functions of one or more independent variables. For example, the speech signal in **Figure 1** can be represented mathematically by a function s(t) that shows the variation of acoustic pressure as a function of time. On the other hand, the independent variable in the mathematical representation of a signal may be either continuous or discrete. Continuous-time signals are defined along a continuum of time and are thus represented by a continuous independent variable. Continuous-time signals are often referred to as analog signals.

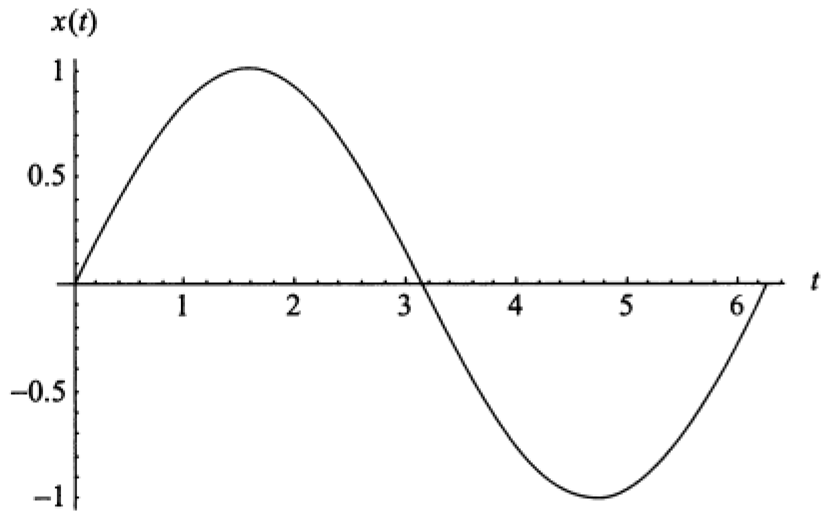


**Figure 3:** An example of a continues time, sine wave

- Discrete-time signals are defined at discrete times, and thus, the independent variable has discrete values; that is, discrete-time signals are represented as sequences of numbers. Digital signals are those for which both time and amplitude are discrete.

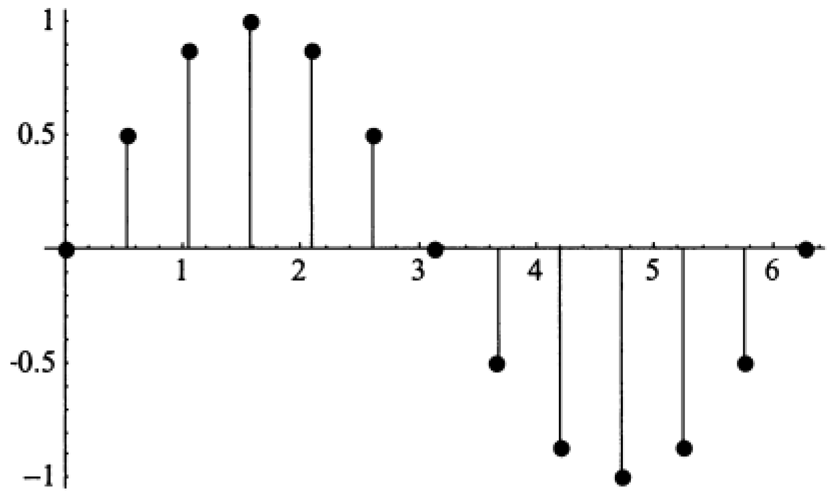


**Figure 4:** A discrete time signal formed by sampling the sine function

- **Signal processing** is a discipline concerned with the acquisition, representation, analysis, manipulation, and transformation of signals. Signal processing involves the physical realization of mathematical operations and it is essential for a tremendous number of practical applications. Some key objectives of signal processing are to improve the quality of a signal or extract useful information from a signal, to separate previously combined signals, and to prepare signals for storage and transmission. Signal processing can be grouped into two classes:

    - Analog Signal Processing
    - Digital Signal Processing

- **Analog signal processing** Since most physical quantities are nonelectric, they should first be converted into an electric signal to allow electronic processing. Analog Signal Processing (ASP) is concerned with the conversion of analog signals into electrical signals by special transducers or sensors and their processing by analog electrical and electronic circuits. The output of the sensor requires some form of conditioning, usually amplification, before it can be processed by the analog signal processor. The parts of a typical analog signal processing system are illustrated in Figure 5

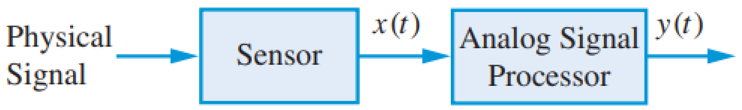


**Figure 5:** Simplified block diagram of an analog signal processing system

- **Digital signal processing** The rapid evolution of digital computing technology which started in the 1960s, marked the transition from analog to digital signal processing. Digital Signal Processing (DSP) is concerned with the representation of analog signals by sequences of numbers, the processing of these sequences by numerical computation techniques, and the conversion of such sequences into analog signals. Digital signal processing has evolved through parallel advances in signal processing theory and the technology that allows its practical application. A typical system for discrete-time processing of continuous-time signals is shown in Figure 6.

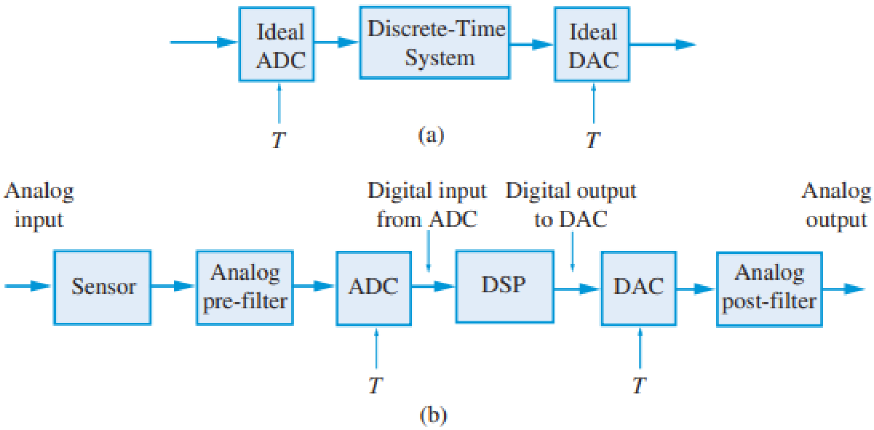


Figure 6: Simplified block diagram of idealized system for (a) continuous-time processing of discrete-time signals, and (b) its practical counterpart for digital processing of analog signals

- In practice, due to inherent real-world limitations, a typical system for the digital processing of analog signals includes the following parts (see Figure 6 (b)): -

1. A sensor that converts the physical quantity to an electrical variable. The output of the sensor is subject to some form of conditioning, usually amplification, so that the voltage of the signal is within the voltage sensitivity range of the converter.

2. An analog filter (known as pre-filter or antialiasing filter) used to “smooth” the input signal before sampling to avoid a serious sampling artifact known as aliasing distortion.

3. An A/D converter that converts the analog signal to a digital signal. After the samples of a discrete-time signal have been stored in memory, time-scale information is lost. The sampling rate and the number of bits used by the ADC determine the accuracy of the system.

4. A digital signal processor (DSP) that executes the signal processing algorithms. The DSP is a computer chip that is similar in many ways to the microprocessor used in personal computers. A DSP is, however, designed to perform certain numerical computations extremely fast. Discrete-time systems can be implemented in real-time or off-line, but ADC and DAC always operate in real-time.

5. A D/A converter that converts the digital signal to an analog signal. The DAC, which reintroduces the lost time-scale information, is usually followed by a sample-and-hold circuit. Usually, the A/D and D/A converters operate at the same sampling rate.

6. An analog filter (known as reconstruction or anti-imaging filter) used to smooth the staircase output of the DAC to provide a more faithful analog reproduction of the digital signal

- **Signal-processing systems** may be classified along the same lines as signals. That is, continuous-time systems are systems for which both the input and the output are continuous-time signals, and discrete-time systems are those for which both the input and the output are discrete-time signals. Based on the type of input and output signal, there are three classes of practical system: analog systems, digital systems, and analog-digital interface systems. The different types of system are summarized in Figure 7

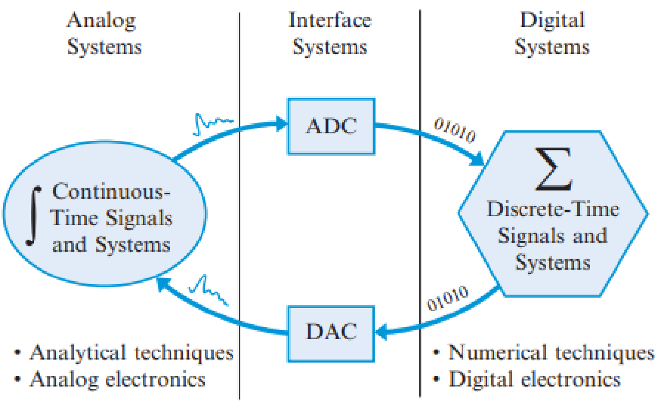


**Figure 7:** The three classes of system: analog systems, digital systems, and interface systems from analog-to-digital and digital-to-analog

- Continuous-time signals are typically processed using analog systems composed of electrical circuit components such as resistors, capacitors, and inductors together with semiconductor electronic components such as diodes, transistors, and operational amplifiers, among others.

- Discrete time signals, on the other hand, are represented mathematically sequences of numbers and processing them requires numerical manipulation of these sequences. Simple addition, multiplication, and delay operations are enough to implement many discrete-time systems. Thus, digital signal processing systems are easier to design, develop, simulate, test, and implement than analog systems by using flexible, reconfigurable, and reliable software and hardware tools. A sequence of numbers x, in which the nth number in the sequence is denoted x[n], is formally written as

$$x = \{x[n]\}, \qquad -\infty < n < \infty,$$

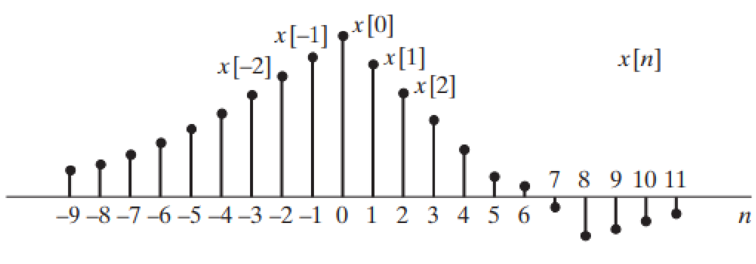


**Figure 8:** Graphic representation of a discrete-time signal.

- Digital signal processing applications often require heavy arithmetic operations, e.g., repeated multiplications and additions, and as such dedicated hardware is required. Possible **implementations** for a real-time implementation of the developed algorithms are:

  - General-purpose microprocessors (μPs) and micro-controllers (μCs).
  - General-purpose digital signal processors (DSPs).
  - Field-programmable gate arrays (FPGAs).

  Selecting the best implementation hardware depends on the requirements of the application such as performance, cost, size, and power consumption.

- Digital signal processing systems are employed these days in many applications such as cell phones, household appliances, cars, ships and airplanes, smart home applications, and many other consumer electronic devices. Table 1 shows the importance of digital signal processing technology in real-world applications.

**Table 1** Examples of digital signal processing applications and algorithms.

| Application area | DSP algorithm |
|---|---|
| Key operations | convolution, correlation, filtering, finite discrete transforms, modulation, spectral analysis, adaptive filtering |
| Audio processing | compression and decompression, equalization, mixing and editing, artificial reverberation, sound synthesis, stereo and surround sound, and noise cancelation |
| Speech processing | speech synthesis, compression and decompression, speech recognition, speaker identification, and speech enhancement |
| Image and video processing | image compression and decompression, image enhancement, geometric transformations, feature extraction, video coding, motion detection, and tomographic image reconstruction |
| Telecommunications (transmission of audio, video, and data) | modulation and demodulation, error detection and correction coding, encryption and decryption, acoustic echo cancelation, multipath equalization, computer networks, radio and television, and cellular telephony |
| Computer systems | sound and video processing, disk control, printer control, modems, internet phone, radio, and television |
| Military systems | guidance and navigation, beamforming, radar and sonar processing, hyperspectral image processing, and software radio |

- Digital signal processing has many **advantages** compared to analog signal processing. The most important are summarized in the following list:

1. Digital programmable systems allow flexibility. DSP programs can be configured by simply making alterations in our program. Reconfiguration of an analog system usually implies a redesign of the hardware.

2. Digital signal processing systems exhibit high accuracy.

3. DSP programs can be stored on magnetic media (disk) without any loss in signal. As a consequence, the signals become portable and can be processed off-line in a remote laboratory.

4. Digital systems are inherently more reliable, more compact, and less sensitive to environmental conditions and component aging than analog systems.

5. Processing in DSP reduces the cost by time-sharing of the processor among a number of different signal processing functions.

6. Digital circuits are less sensitive to tolerance of a component value.

6. The implementation of highly sophisticated signal processing algorithms is made possible with DSP. It is very difficult to perform precise mathematical operations on signals in the analog form.

- **Audio Signal processing** is a method where intensive algorithms, techniques are applied to audio signals. Audio signals are the representation of sound, which is in the form of digital and analog signals. Their frequencies range between 20 to 20,000 Hz, and this is the lower and upper limit of our ears. Analog signals occur in electrical signals, while digital signals occur in binary representations. This process encompasses removing unwanted noise and balancing the time-frequency ranges by converting digital and analog signals.

It focuses on computational methods for processing the sounds. It removes or minimizes the overmodulation, echo, unwanted noise by applying various techniques. Digitizing audio waves refers to converting analog sound waves into digital signals that a machine can store and manipulate. This process involves several steps:

- **Sampling**: The analog sound wave is measured at regular intervals, and each measurement is assigned a numerical value. The rate at which these measurements are taken is called the **sampling rate** and is typically measured in kilohertz (kHz).

- **Quantization**: The numerical values obtained through sampling are then rounded to the nearest whole number. The number of bits used for quantization determines the dynamic range of the digital signal or the range between the quietest and loudest sounds that can be represented.

- **Encoding**: The quantized values are then encoded into a digital format, such as a WAV or MP3 file, that can be stored on a computer or other digital storage media.

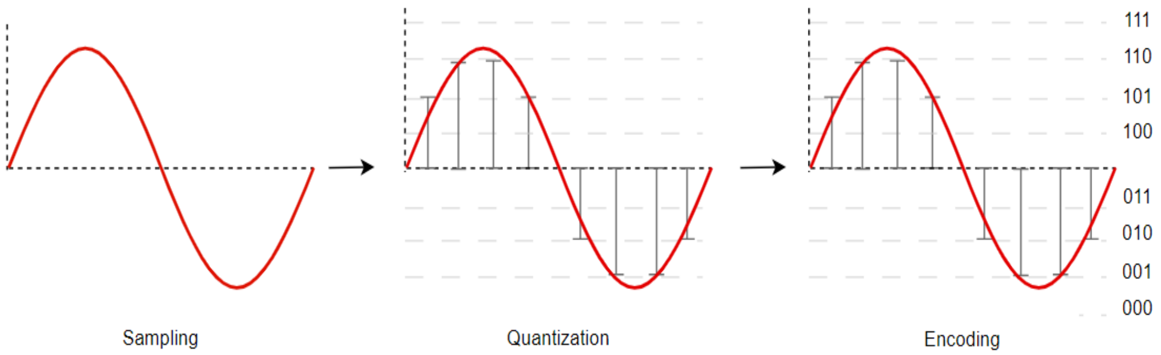


**Figure 9:** Analog to Digital Conversion of a Sound Signal

The resulting digital signal that is presented in **Figure 9** can be manipulated in various ways, such as edited, mixed with other audio signals, or played back through speakers or headphones.

- **Acoustic Features**

An audio clip that is to be processed by machine learning algorithms for predictions needs descriptors because the raw digitized form will not provide the necessary information for the model to learn patterns hidden in speech. These patterns are known as acoustic features. There are two primary categories of acoustic features: prosodic and spectral. Prosodic features depict speech's rhythmic, intonational, and stress-related aspects. These features include the pitch, the intonation, and the speed of speech. On the other hand, spectral features pertain to the energy distribution across different frequencies. These features can provide information about the quality or timbre of the sound, as well as other characteristics like pitch and loudness. Analyzing the frequencies in the vocal tract using spectral features can provide details about the speaker's gender, age, and other attributes.

- **Example: Speech Emotion Recognition**

After extracting the sound features, the next step is to identify a speech-processing task and feed these acoustic features to train a model. Consider the problem of speech emotion recognition, where the model determines the emotion in an audio clip. Speech can be classified into four basic emotions: happy, sad, neutral, and angry. The following illustration shows how speech descriptors can be used to train a model:

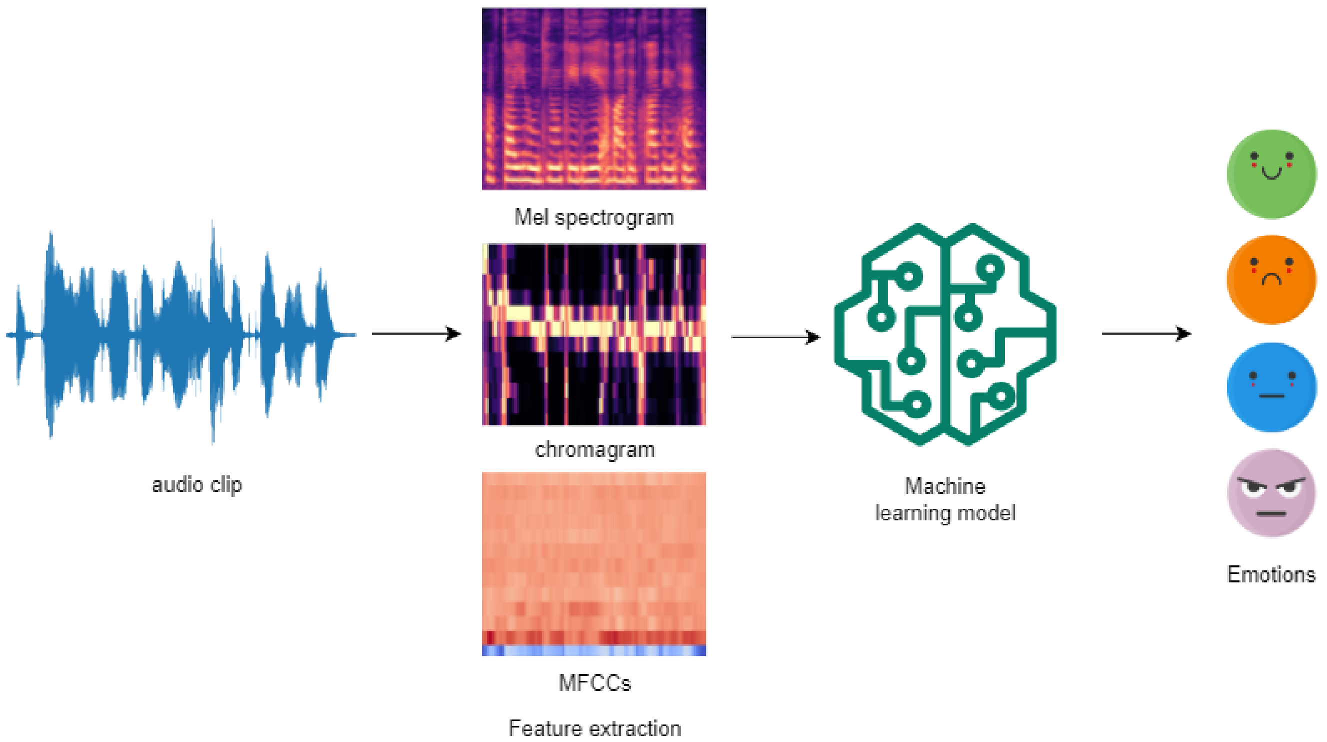


**Figure 10:** Speech Emotion Recognition

## • Elementary Signals in DSP

There are several elementary signals which play vital role in the study of signal processing. These elementary signals serve as basic building blocks for the construction of more complex signals. In fact, these elementary signals may be used to model a large number of physical signals, which occur in nature. These elementary signals are also called standard signals.

The standard signals are as follows:

1. Unit step Function
2. Unit ramp Function
3. Unit parabolic Function
4. Unit impulse Function
5. Sinusoidal Function
6. Real exponential Function
7. Complex exponential Function

### 1. The Unit Step Function

The step Function is an important signal used for analysis of many continuous and discrete-time systems. It exists only for positive time and is zero for negative time. It is equivalent to applying a signal whose amplitude suddenly changes and remains constant at the sampling instants forever after application. In between the discrete instants it is zero. If a step function has unity magnitude, then it is called unit step function. The usefulness of the unit-step function lies in the fact that if we want a signal to start at $t = 0$, so that it may have a value of zero for $t < 0$, we only need to multiply the given signal with unit step function $u(t)$.

The continuous-time unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

From the above equation for $u(t)$, we can observe that when the argument $t$ in $u(t)$ is less than zero, then the unit step function is zero, and when the argument $t$ in $u(t)$ is greater than or equal to zero, then the unit step function is unity.

The shifted unit step function $u(t-a)$ is defined as:

$$u(t-a) = \begin{cases} 1 & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

It is zero if the argument $(t-a) < 0$ and equal to 1 if the argument $(t-a) \geq 0$.
The graphical representations of $u(t)$ and $u(t-a)$ are shown in Figure 1.[(a) and (b)].

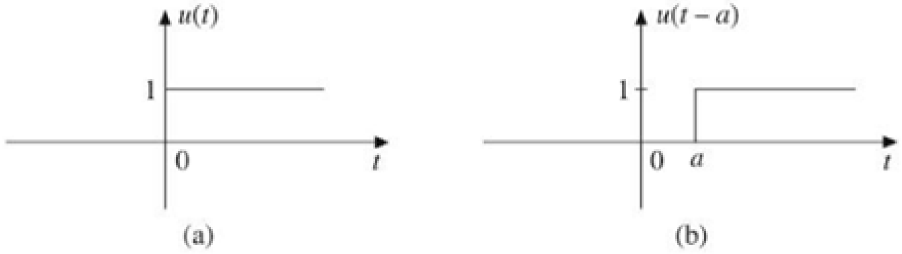


**Figure 11** (a) Unit step function, (b) Delayed unit step function.

The discrete-time unit step sequence $u(n)$ is defined as:

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

The shifted version of the discrete-time unit step sequence $u(n-k)$ is defined as

$$u(n-k) = \begin{cases} 1 & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

The graphical representations of $u(n)$ and $u(n-k)$ are shown in **Figure** 2[(a) and (b)].

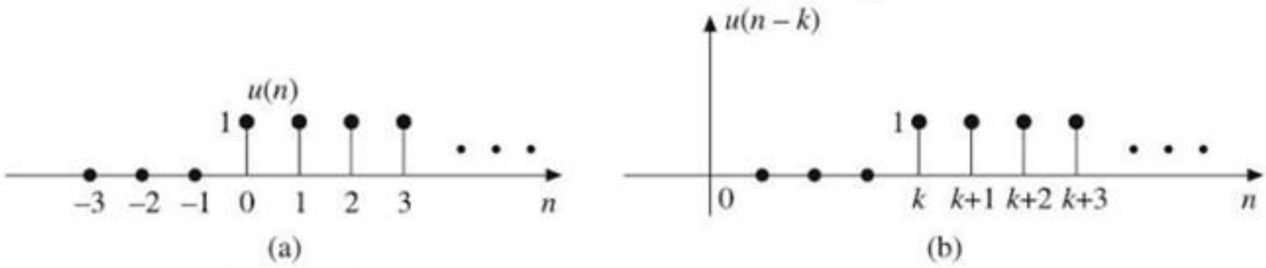


**Figure 12:** Discrete-time (a) Unit step function, (b) Shifted unit step function.

## 2. Unit Ramp Function

The continuous-time unit ramp sequence r(t) is that sequence which starts at t = 0 and increases linearly with time and is defined as:

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or $$r(t) = t\,u(t)$$

The unit ramp function has unit slope. It is a signal whose amplitude varies linearly. It can be obtained by integrating the unit step function. That means, a unit step signal can be obtained by differentiating the unit ramp signal.

i.e. $$r(t) = \int u(t)\,dt = \int dt = t \quad \text{for } t \geq 0$$

$$u(t) = \frac{d}{dt} r(t)$$

The delayed unit ramp signal $r(t-a)$ is given by

$$r(t-a) = \begin{cases} t-a & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

or $$r(t-a) = (t-a)\,u(t-a)$$

The graphical representations of $r(t)$ and $r(t-a)$ are shown in Figure 3 [(a) and (b)].

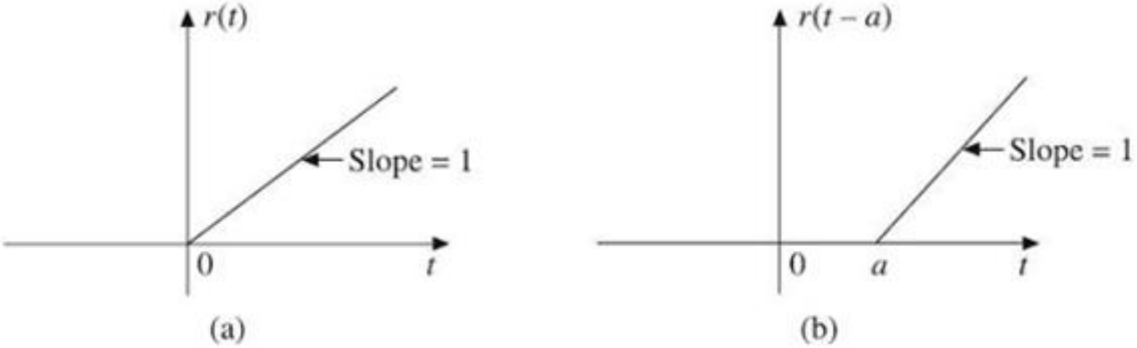


**Figure 13:** (a) Unit ramp signal, (b) Delayed unit ramp signal.

The discrete-time unit ramp sequence $r(n)$ is defined as

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or $$r(n) = n\,u(n)$$

The shifted version of the discrete-time unit-ramp sequence $r(n-k)$ is defined as

$$r(n-k) = \begin{cases} n-k & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

or $$r(n-k) = (n-k)\,u(n-k)$$

The graphical representations of $r(n)$ and $r(n-2)$ are shown in Figure 4 [(a) and (b)].

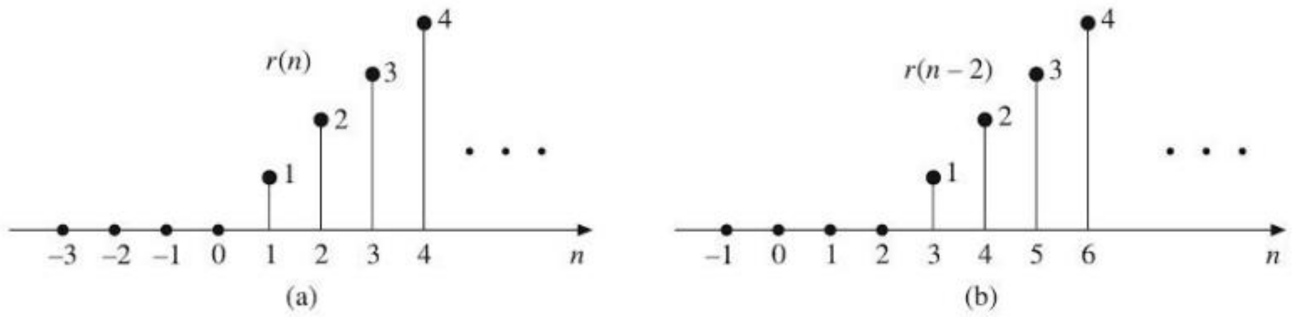


Figure 4 Discrete-time (a) Unit-ramp sequence, (b) Shifted-ramp sequence.

## 3. Unit Parabolic Function

The continuous-time unit parabolic function $p(t)$, also called unit acceleration signal starts at $t = 0$, and is defined as:

$$p(t) = \begin{cases} \dfrac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or
$$p(t) = \frac{t^2}{2} u(t)$$

The shifted version of the unit parabolic sequence $p(t-a)$ is given by

$$p(t-a) = \begin{cases} \dfrac{(t-a)^2}{2} & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

or
$$p(t-a) = \frac{(t-a)^2}{2} u(t-a)$$

The graphical representations of $p(t)$ and $p(t-a)$ are shown in Figure 5 [(a) and (b)].

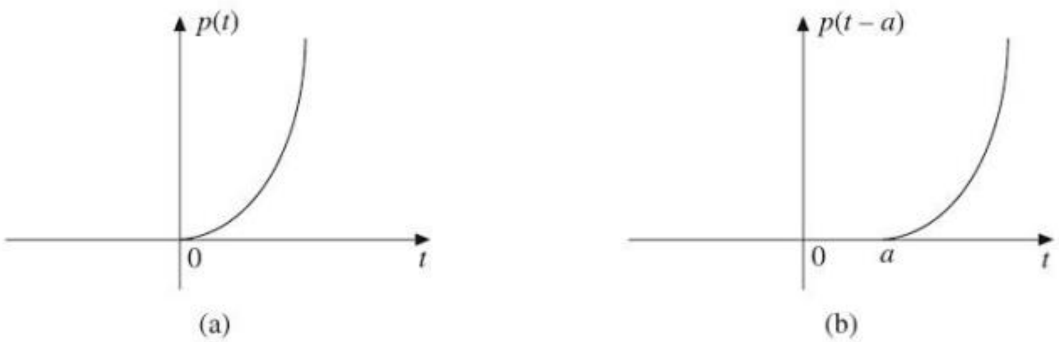


Figure 5 (a) Unit parabolic signal, (b) Delayed parabolic signal.

The unit parabolic function can be obtained by integrating the unit ramp function or double integrating the unit step function.

$$p(t) = \iint u(t)\, dt = \int r(t)\, dt = \int t\, dt = \frac{t^2}{2} \quad \text{for } t \geq 0$$

The ramp function is derivative of parabolic function and step function is double derivative of parabolic function

$$r(t) = \frac{d}{dt}\,p(t); \quad u(t) = \frac{d^2}{dt^2}\,p(t)$$

The discrete-time unit parabolic sequence $p(n)$ is defined as:

$$p(n) = \begin{cases} \frac{n^2}{2} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or

$$p(n) = \frac{n^2}{2}\,u(n)$$

The shifted version of the discrete-time unit parabolic sequence $p(n-k)$ is defined as:

$$p(n-k) = \begin{cases} \frac{(n-k)^2}{2} & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

or

$$p(n-k) = \frac{(n-k)^2}{2}\,u(n-k)$$

The graphical representations of $p(n)$ and $p(n-3)$ are shown in Figure 6 [(a) and (b)].

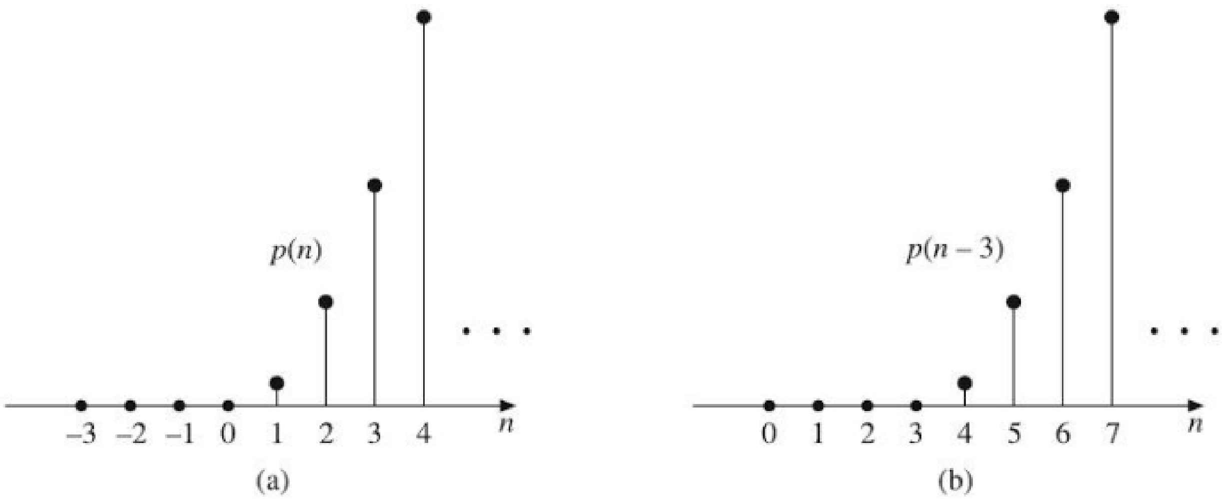


Figure 6 Discrete-time (a) Parabolic sequence, (b) Shifted parabolic sequence.

## 4. Unit Impulse Function

The unit impulse function is the most widely used elementary function used in the analysis of signals and systems. The continuous-time unit impulse function $\delta(t)$, also called Dirac delta function, plays an important role in signal analysis. It is defined as:

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases},$$

and the above Equation is also constrained to satisfy the identity as

$$\int_{-\infty}^{+\infty} \delta(t)\mathrm{d}t = 1$$

That is, the impulse function has zero amplitude everywhere except at $t = 0$. At $t = 0$, the amplitude is infinity so that the area under the curve is unity. $\delta(t)$ can be represented as a limiting case of a rectangular pulse function.

As shown in Figure 7 (a),

$$x(t) = \frac{1}{\Delta}[u(t) - u(t-\Delta)]$$

$$\delta(t) = \operatorname*{Lt}_{\Delta \to 0} x(t) = \operatorname*{Lt}_{\Delta \to 0} \frac{1}{\Delta}[u(t) - u(t-\Delta)]$$

A delayed unit impulse function $\delta(t - a)$ is defined as:

$$\delta(t-a) = \begin{cases} 1 & \text{for } t = a \\ 0 & \text{for } t \neq a \end{cases}$$

The graphical representations of $\delta(t)$ and $\delta(t - a)$ are shown in Figure 7 [(b) and (c)].

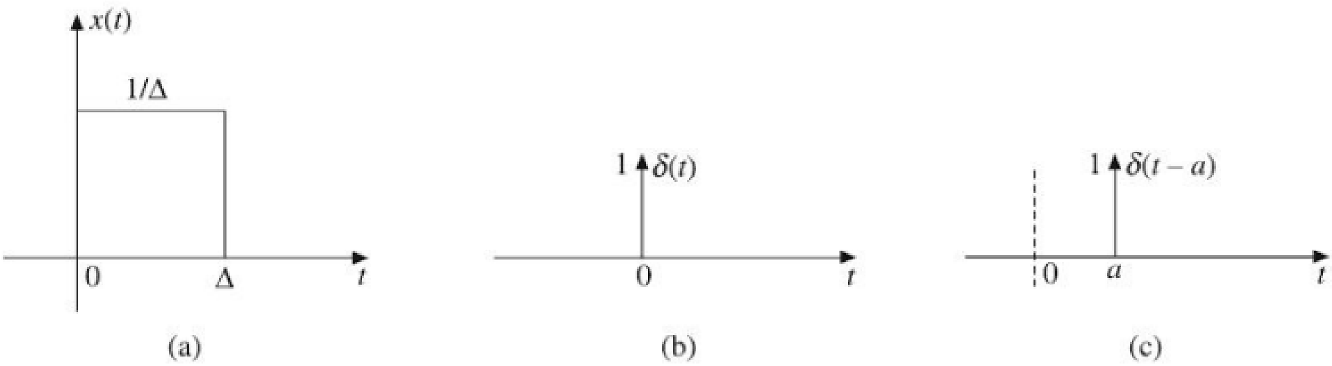


**Figure 7** (a) $\delta(t)$ as limiting case of a pulse, (b) Unit impulse, (c) Delayed unit impulse.

If unit impulse function is assumed in the form of a pulse, then the following points may be observed about a unit impulse function.

(i) The width of the pulse is zero. This means the pulse exists only at $t = 0$.
(ii) The height of the pulse goes to infinity.
(iii) The area under the pulse curve is always unity.
(iv) The height of arrow indicates the total area under the impulse.

The integral of unit impulse function is a unit step function and the derivate of unit step function is a unit impulse function.

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt$$

and

$$\delta(t) = \frac{d}{dt} u(t)$$

***Properties of continuous-time unit impulse function***

1. It is an even function of time $t$, i.e. $\delta(t) = \delta(-t)$
2. $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$
3. $\delta(at) = \dfrac{1}{|a|}\delta(t)$
4. $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0)$; $x(t)\,\delta(t) = x(0)\,\delta(t) = x(0)$
5. $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

The discrete-time unit impulse function $\delta(n)$, also called unit sample sequence, is defined as:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

The shifted unit impulse function $\delta(n - k)$ is defined as:

$$\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$$

The graphical representations of $\delta(n)$ and $\delta(n - 3)$ are shown in Figure 8 [(a) and (b)].


**Figure 8** Discrete-time (a) Unit sample sequence, (b) Delayed unit sample sequence.

*Properties of discrete-time unit sample sequence*

1. $\delta(n) = u(n) - u(n-1)$
2. $\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$
3. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$
4. $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

***Relation between the unit sample sequence and the unit step sequence***

The unit sample sequence $\delta(n)$ and the unit step sequence $u(n)$ are related as:

$$u(n) = \sum_{m=0}^{n} \delta(m), \quad \delta(n) = u(n) - u(n-1)$$

## 5. Sinusoidal Signal

A continuous-time sinusoidal signal in its most general form is given by

$$x(t) = A \sin(\omega t + \phi)$$

where

$A$ = Amplitude
$\omega$ = Angular frequency in radians
$\phi$ = Phase angle in radians

Figure 9 shows the waveform of a sinusoidal signal. A sinusoidal signal is an example of a periodic signal. The time period of a continuous-time sinusoidal signal is given by

$$T = \frac{2\pi}{\omega}$$

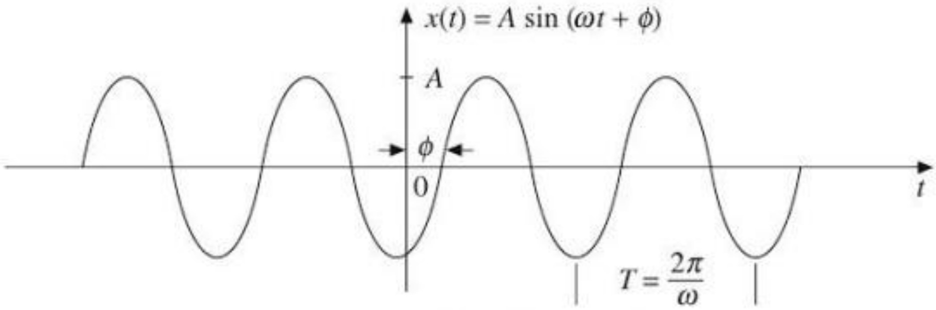


Figure 9 Sinusoidal waveform.

The discrete-time sinusoidal sequence is given by

$$x(n) = A \sin(\omega n + \phi)$$

where $A$ is the amplitude, $\omega$ is angular frequency, $\phi$ is phase angle in radians and $n$ is an integer.

The period of the discrete-time sinusoidal sequence is:

$$N = \frac{2\pi}{\omega} m$$

where $N$ and $m$ are integers.

*All continuous-time sinusoidal signals are periodic but discrete-time sinusoidal sequences may or may not be periodic depending on the value of $\omega$.*

For a discrete-time signal to be periodic, the angular frequency $\omega$ must be a rational multiple of $2\pi$.

The graphical representation of a discrete-time sinusoidal signal is shown in Figure **10**

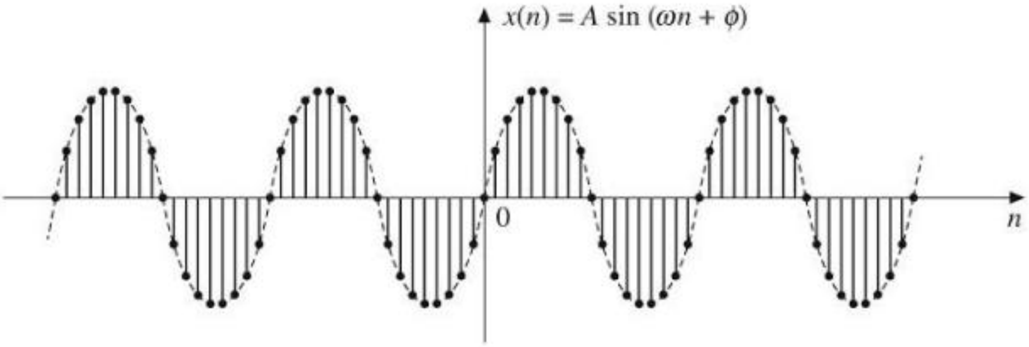


Figure **10** Discrete-time sinusoidal signal.

## 6. Real Exponential Signal

A continuous-time real exponential signal has the general form as:

$$x(t) = Ae^{\alpha t}$$

where both $A$ and $\alpha$ are real.

The parameter $A$ is the amplitude of the exponential measured at $t = 0$. The parameter $\alpha$ can be either positive or negative. Depending on the value of $\alpha$, we get different exponentials.

1. If $\alpha = 0$, the signal $x(t)$ is of constant amplitude for all times.
2. If $\alpha$ is positive, i.e. $\alpha > 0$, the signal $x(t)$ is a growing exponential signal.
3. If $\alpha$ is negative, i.e. $\alpha < 0$, the signal $x(t)$ is a decaying exponential signal.

These three waveforms are shown in Figure **11** [(a), (b) and (c)].

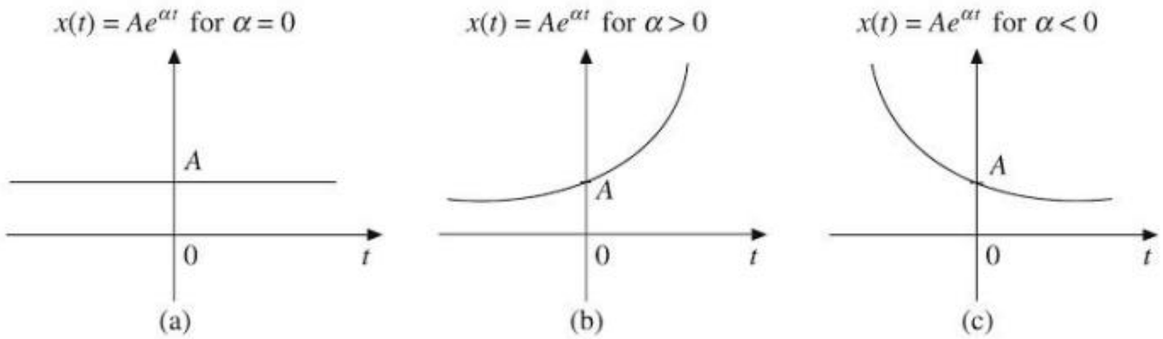


Figure **11** Continuous-time real exponential signals $x(t) = Ae^{\alpha t}$ for (a) $\alpha = 0$, (b) $\alpha > 0$, (c) $\alpha < 0$

The discrete-time real exponential sequence $a^n$ is defined as:

$$x(n) = a^n \quad \text{for all } n$$

Figure12 illustrates different types of discrete-time exponential signals.

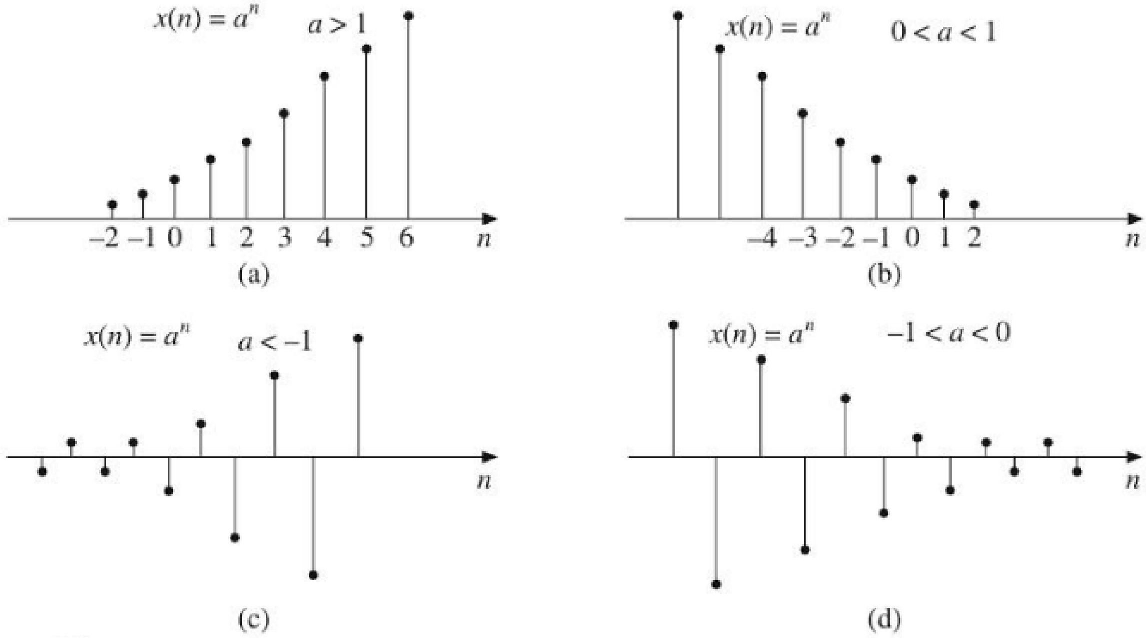


Figure12 Discrete-time exponential signal $a^n$ for (a) $a > 1$, (b) $0 < a < 1$, (c) $a < -1$, (d) $-1 < a < 0$.

## 7. Complex Exponential Signal

The complex exponential signal has a general form as

$$x(t) = Ae^{st}$$

where $A$ is the amplitude and $s$ is a complex variable defined as

$$s = \sigma + j\omega$$

Therefore,
$$x(t) = Ae^{st} = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t}e^{j\omega t}$$
$$= Ae^{\sigma t}[\cos \omega t + j \sin \omega t]$$

Depending on the values of $\sigma$ and $\omega$, we get different waveforms as shown in Figure13

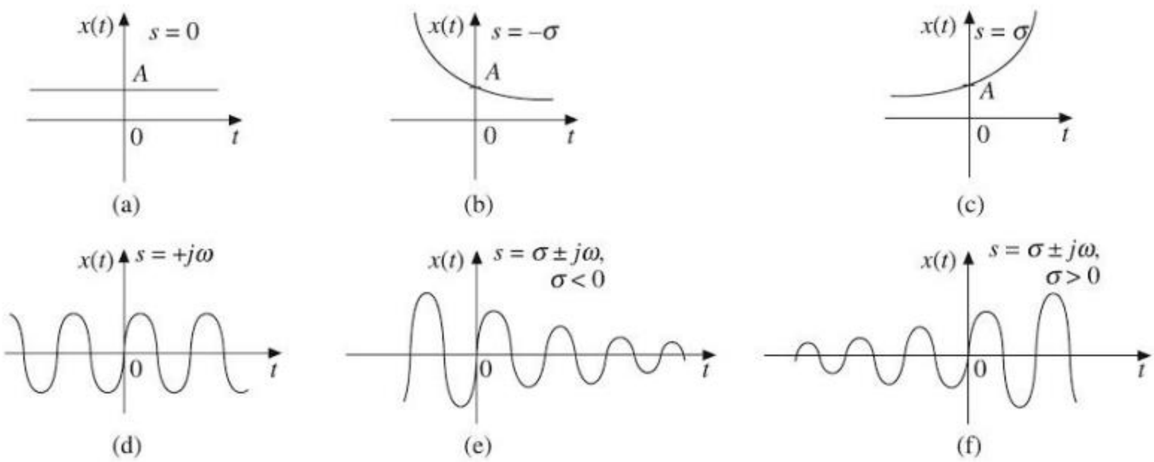


Figure13 Complex exponential signals.

The discrete-time complex exponential sequence is defined as

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$
$$= a^n \cos(\omega_0 n + \phi) + ja^n \sin(\omega_0 n + \phi)$$

For $|a| = 1$, the real and imaginary parts of complex exponential sequence are sinusoidal.
For $|a| > 1$, the amplitude of the sinusoidal sequence exponentially grows
For $|a| < 1$, the amplitude of the sinusoidal sequence exponentially decays

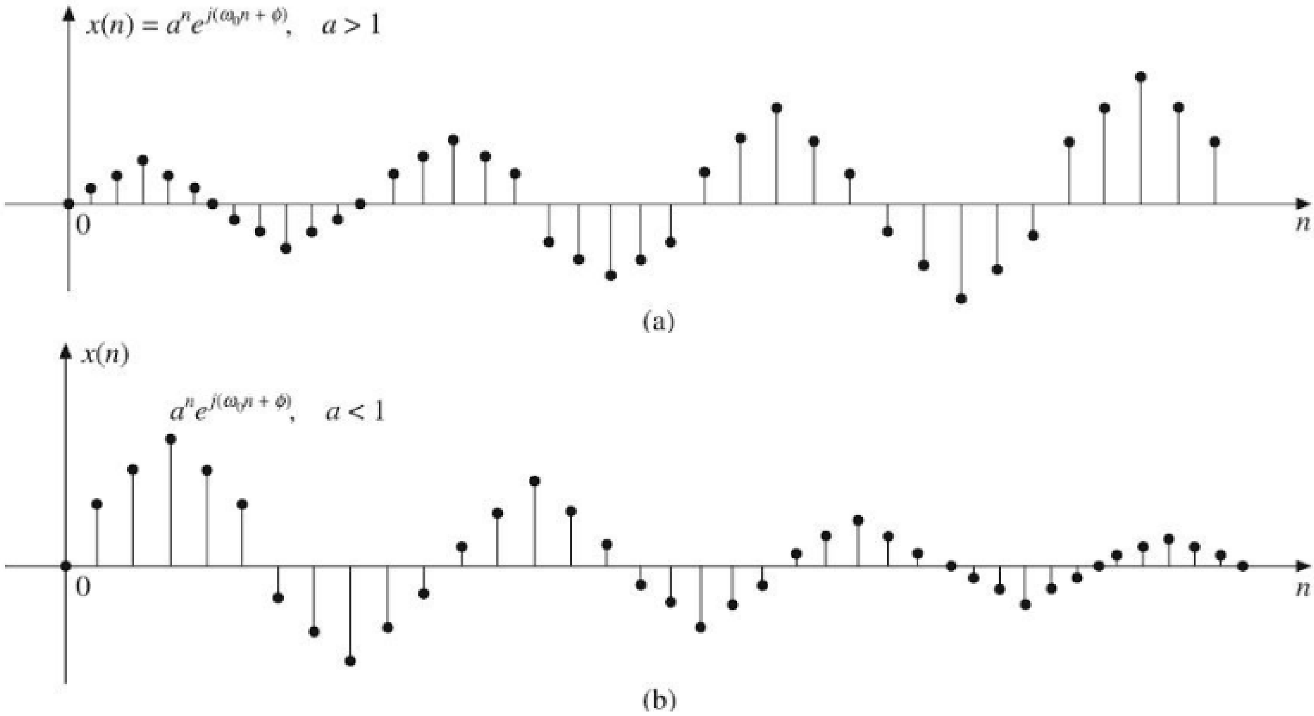


**Figure 14** Complex exponential sequence $x(n) = a^n e^{j(\omega_0 n + \phi)}$ for (a) $a > 1$, (b) $a < 1$.