Example: In a company the distribution of employees ages and their salary as follows

Age	X	20	25	30	35	40	45	50	60	65
Salary	Y	2.5	3	3	3.5	3.5	4	4.5	4.5	5

If you have the fuzzy set $A_1 = \{$young employees$\}$ that defined on X, then find the fuzzy set B_1 defined on Y s.t $B_1 = \{$young employees salary$\}$, $B_1 = f(A)$ and if you have a fuzzy set $B_2 = \{$low salary$\}$ defined on Y, then find the fuzzy set $A_2 = \{$employees with low salary$\}$ defined on X.

$$\mu_{A_1}(x) = \begin{cases} 1 & ; \; x = 20 \\ 0.8 & ; \; x = 25, 30 \\ 0.6 & ; \; x = 35 \\ 0.4 & ; \; x = 40 \\ 0.2 & ; \; x = 45 \\ 0 & ; \; x = 50, 60, 65 \end{cases}$$

$$\mu_{B_2}(y) = \begin{cases} 1 & ; \; y = 2.5 \\ 0.75 & ; \; y = 3 \\ 0.5 & ; \; y = 3.5 \\ 0.25 & ; \; y = 4 \\ 0 & ; \; y = 4.5, 5 \end{cases}$$

Solution : $X = \{20, 25, 30, 35, 40, 45, 50, 60, 65\}$

$Y = \{2.5, 3, 3.5, 4, 4.5, 5\}$

$f(20) = 2.5$, $f(25) = 3$, $f(30) = 3$, $f(35) = 3.5$, $f(40) = 3.5$, $f(45) = 4$

$f(50) = 4.5$, $f(60) = 4.5$, $f(65) = 5$

$B_1 = \{$ young employees salary $\}$, this fuzzy set depended on A_1 and the function f,

$$\mu_{B_1}(y) = \sup_{x/y=f(x)} \mu_{A_1}(x)$$

$$B_1 = \{ (\mu_{B_1}(y), y) : y \in Y , y = f(x), \forall x \in X \}$$

when $y = 2.5$, $\mu_{B_1}(2.5) = \sup_{2.5 = f(20)} \{ \mu_{A_1}(20) \} = \sup \{1\} = 1$

$\Rightarrow \mu_{B_1}(2.5) = 1$

when $y = 3$, $\mu_{B_1}(3) = \sup_{\substack{3 = f(25) \\ 3 = f(30)}} \{ \mu_{A_1}(25), \mu_{A_1}(30) \} = \sup \{0.8, 0.8\}$

$\Rightarrow \mu_{B_1}(3) = 0.8$

when $y = 3.5$, $\mu_{B_1}(3.5) = \sup_{\substack{3.5 = f(35) \\ 3.5 = f(40)}} \{ \mu_{A_1}(35), \mu_{A_1}(40) \} = \sup \{0.6, 0.4\}$

$\Rightarrow \mu_{B_1}(3.5) = 0.6$

when $y=4$, $\mu_{B_1}(4) = \sup\{\mu_{A_1}(45)\} = \sup\{0.2\} \Rightarrow \mu_{B_1}(4) = 0.2$

$4 = f(45)$

when $y = 4.5$, $\mu_{B_1}(4.5) = \sup\{\mu_{A_1}(50), \mu_{A_1}(60)\} = \sup\{0, 0\} \Rightarrow$

$4.5 = f(50)$

$4.5 = f(60)$

$\mu_{B_1}(4.5) = 0$

when $y = 5$, $\mu_{B_1}(5) = \sup\{\mu_{A_1}(65)\} = \sup\{0\}$

$5 = f(65)$

$\Rightarrow \mu_{B_1}(5) = 0$

$$\mu_{B_1}(y) = \begin{cases} 1 & , 2.5 \\ 0.8 & ; 3 \\ 0.6 & , 3.5 \\ 0.2 & , 4 \\ 0 & ; 4.5, 5 \end{cases}$$

Now, since $B_2 = \{$low salary$\}$ defined on Y then

$$A_2 = \{(\mu_{A_2}(x), x), \ \mu_{A_2}(x) = \mu_{B_2}(f(x)), \ x \in X\}$$

$$\mu_{A_2 = f^{-1}(B_2)}(x) = \mu_{B_2}(f(x)), \quad \mu_{A_2}(20) = \mu_{B_2}(f(20)) = \mu_{B_2}(2.5) = 1$$

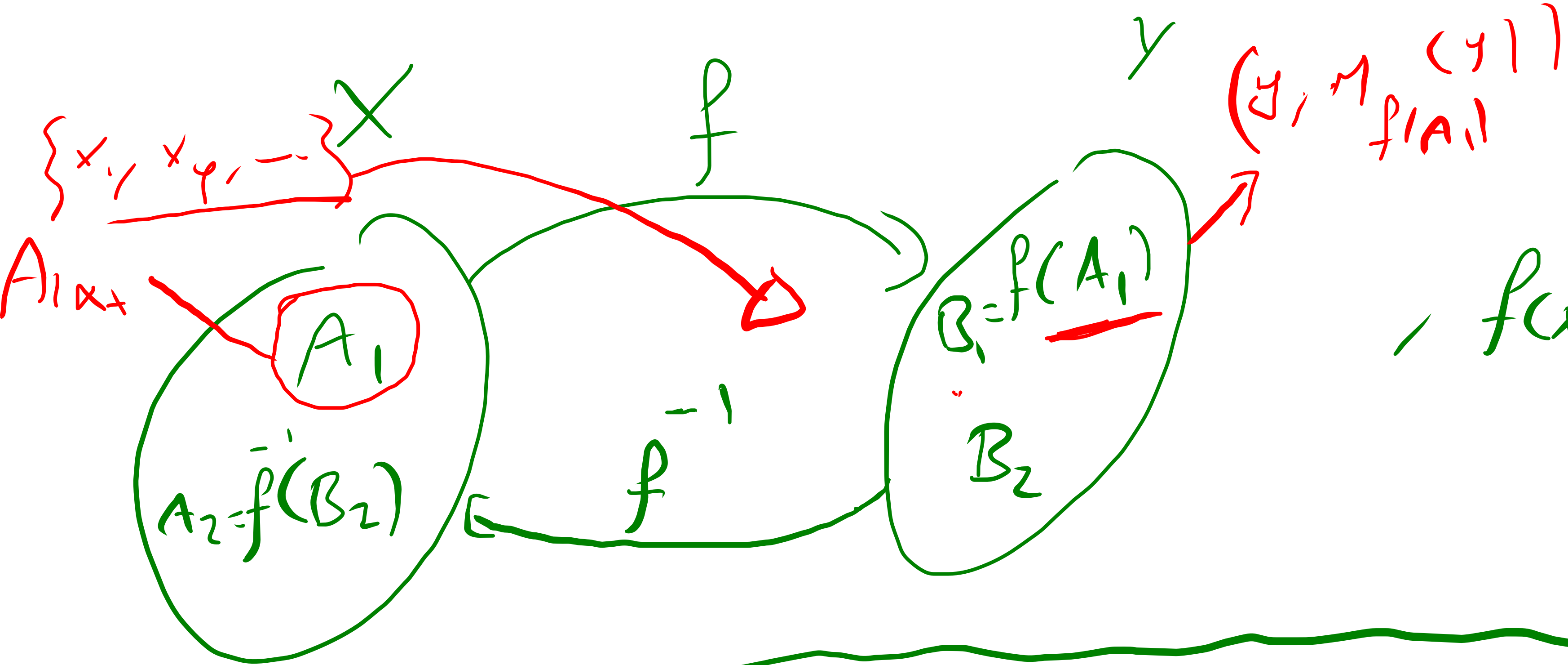
$$\Rightarrow \mu_{A_2}(20) = 1$$

$$x = 25, \ \mu_{A_2}(25) = \mu_{B_2}(f(25)) = \mu_{B_2}(3) = 0.75 \Rightarrow \mu_{A_2}(25) = 0.75$$

$$x = 30, \ \mu_{A_2}(30) = \mu_{B_2}(f(30)) = \mu_{B_2}(3) = 0.75 \Rightarrow \mu_{A_2}(30) = 0.75$$

$$\mu_{A_2}(35) = 0.5, \ \mu_{A_2}(40) = 0.5, \ \mu_{A_2}(45) = 0.25, \ \mu_{A_2}(50) = 0$$

$$\mu_{A_2}(60) = \mu_{A_2}(65) = 0$$



, $f(x) = x^2$

Theorem:- let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in \mathcal{F}(X)$ and all $\alpha \in [0,1]$, then the following properties of f fuzzified by extension principle hold:

① $[f(A)]_{\alpha^+} = f(A_{\alpha^+})$ ② $f(A_\alpha) \subseteq [f(A)]_\alpha$