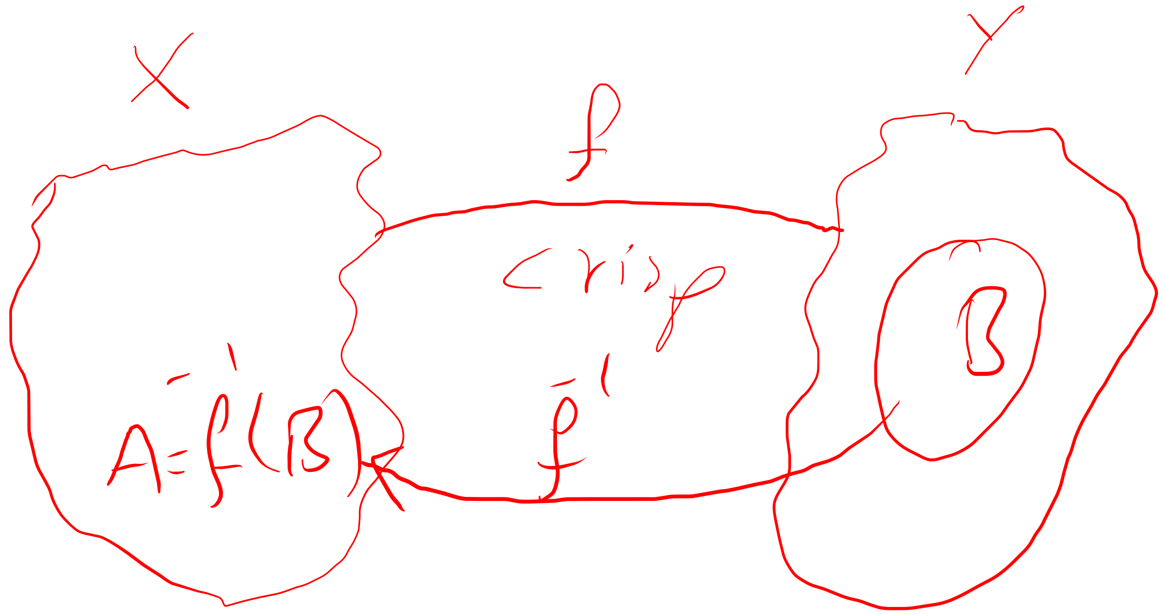
Extension principle for fuzzy sets



$$f(x) = y \,, \quad \forall x \in X \,, \quad y \in Y$$

$A \in \mathcal{F}(X)$, $B \in \mathcal{F}(Y)$, $\mu(y) =$

$B = f(A)$

$B = f(A) \in \mathcal{F}(X)$

$\mu(x) =$

$A = f^{-1}(B)$

Extension principle for fuzzy sets

Let $f : X \longrightarrow Y$ be a crisp function from the crisp set X to a crisp set Y, if we are extended to act fuzzy sets. defined on X and Y, then we say that the crisp function is fuzzified. So the fuzzified function has the form

$$f : \mathcal{F}(X) \longrightarrow \mathcal{F}(Y) \text{ and its inverse function}$$

$$f^{-1} : \mathcal{F}(Y) \longrightarrow \mathcal{F}(X)$$

A principle for fuzzy crisp function is called extension principle.

if we have a crisp function f from X to Y, then its extended version is a function from $P(X)$ to $P(Y)$

that is for any set $A \in P(X)$, then

$$f(A) = \{y : y = f(x), x \in A\} \quad (1)$$

and extended version of inverse of f, denoted by f^{-1} which is a function from $P(Y)$ to $P(X)$ which defined as follow

$$f^{-1}(B) = \{x : f(x) \in B\} \quad (2)$$

if we expressing about (1) and (2) by thier characteristic function (we know that it is a special case of membership function)

$$M_{f(A)}(y) = \sup_{x / y = f(x)} M_A(x) \quad \text{for all } A \in \mathcal{F}(X)$$

$$M_{f^{-1}(B)}(x) = M_B(f(x)) \quad \text{for all } B \in \mathcal{F}(Y)$$

Example :- let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ and let f be a function from X to Y s.t $f(a) = f(b) = 1$ & $f(c) = 2$, so we will show how you can extended verision of the function f from $P(X)$ to $P(Y)$ and its inverse.

Solution : $P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

$P(Y) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$f : P(X) \to P(Y)$, $f(A) = \{y : y = f(x), x \in A\}$

$f(\phi) = \{y : y = f(x), x \in \phi\} = \phi$

$f(\{a\}) = \{y : y = f(x), x \in \{a\}\} = \{1\}$, $f(\{b\}) = \{y : y = f(x), x \in \{b\}\} = \{1\}$

$f(\{c\}) = \{2\}$, $f(\{a, b\}) = \{y : y = f(x), x \in \{a, b\}\} = \{1\}$

$f(\{a, c\}) = \{y : y = f(x), x \in \{a, c\}\} = \{1, 2\}$, $f(\{a, b, c\}) = \{1, 2\}$, $f(\{b, c\}) = \{1, 2\}$

Now, $f^{-1}: P(Y) \to P(X)$, $f^{-1}(B) = \{x : f(x) \in B\}$

$f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{1\}) = \{x : f(x) \in \{1\}\} = \{a, b\}$

$f^{-1}(\{2\}) = \{x : f(x) \in \{2\}\} = \{c\}$, $f^{-1}(\{1,2\}) = \{x : f(x) \in \{1,2\}\}$

$\Rightarrow f^{-1}(\{1,2\}) = \{a, b, c\}$

Extension principle :- Any given function $f: X \to Y$ induces two functions $f: \mathcal{F}(X) \to \mathcal{F}(Y)$ and $f^{-1}: \mathcal{F}(Y) \to \mathcal{F}(X)$ that is defined by

$$\mu_{B=f(A)}(y) = \sup_{x \mid y=f(x)} \mu_A(x) \qquad \forall\, A \in \mathcal{F}(X)$$

$$\mu_{A=f^{-1}(B)}(x) = \mu_B(f(x)) \qquad \forall\, B \in \mathcal{F}(Y)$$