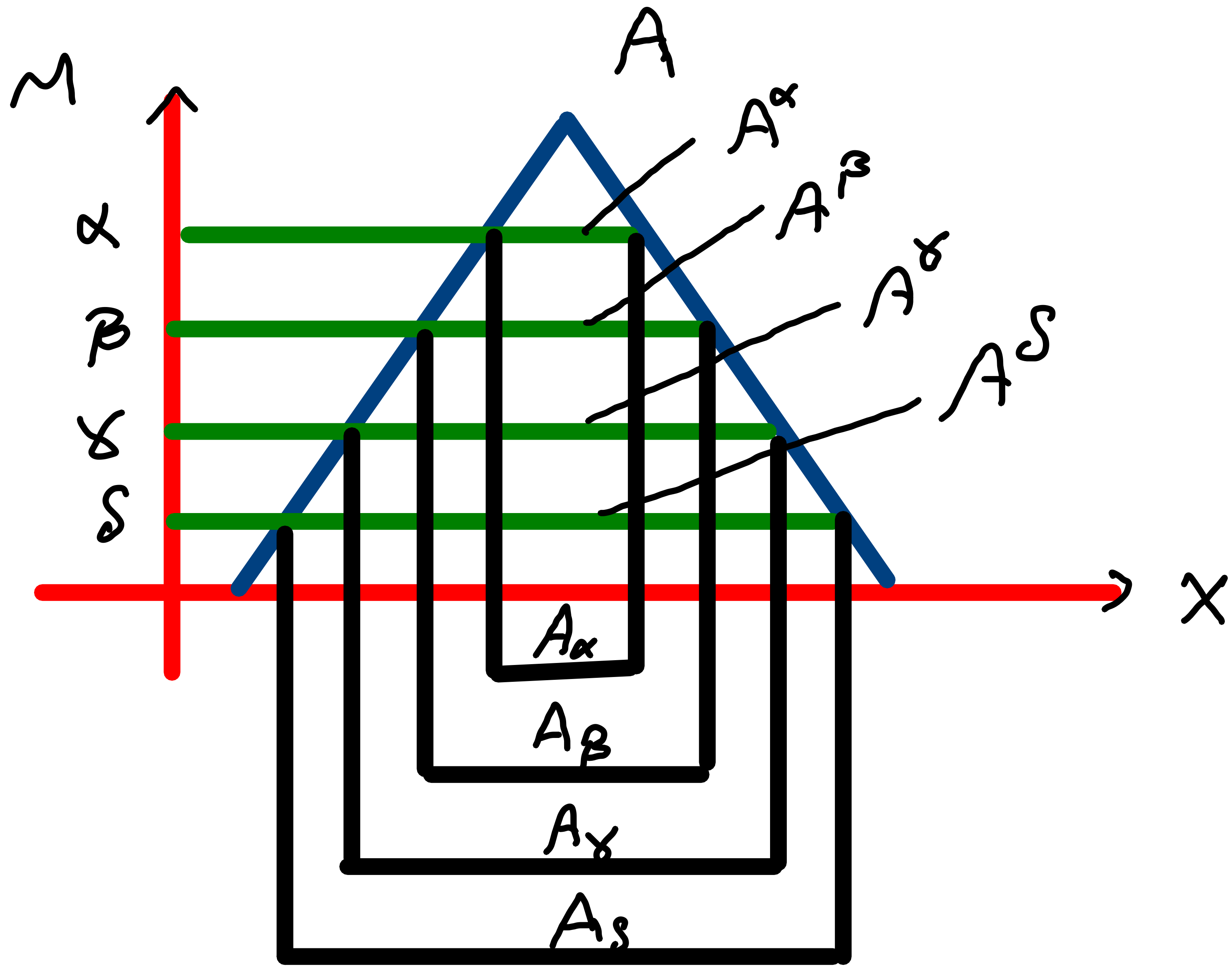


The above figure shows that the special fuzzy set of A which denoted by A^α

Theorem :- (The first Decomposition Theorem)

For every $A \in \mathcal{F}(X)$, Then $A = \bigcup_{\alpha \in [0,1]} {}^{\alpha}A$, ${}^{\alpha}A = \alpha . A_{\alpha}$

proof :- for each particular $x \in X$, let $\mu_A(x) = \beta$

then $\mu_{\bigcup_{\alpha \in [0,1]} {}^{\alpha}A}(x) = \sup_{\alpha \in [0,1]} \mu_{{}^{\alpha}A}(x)$

$$= \max \{ \sup_{\alpha \in [0,\beta]} \mu_{{}^{\alpha}A}(x), \sup_{\alpha \in (\beta,1]} \mu_{{}^{\alpha}A}(x)$$

For each $\alpha \in (\beta, 1]$, we have $\mu_A(x) = \beta < \alpha$ and therefore $\mu_{{}^{\alpha}A}(x) = 0$, on the other hand, for each $\alpha \in [0,\beta]$, we have $\mu_A(x) = \beta \geq \alpha$ and then $\mu_{{}^{\alpha}A}(x) = \alpha$ then

then $\mu_{A^\alpha}(x) = \alpha$, then

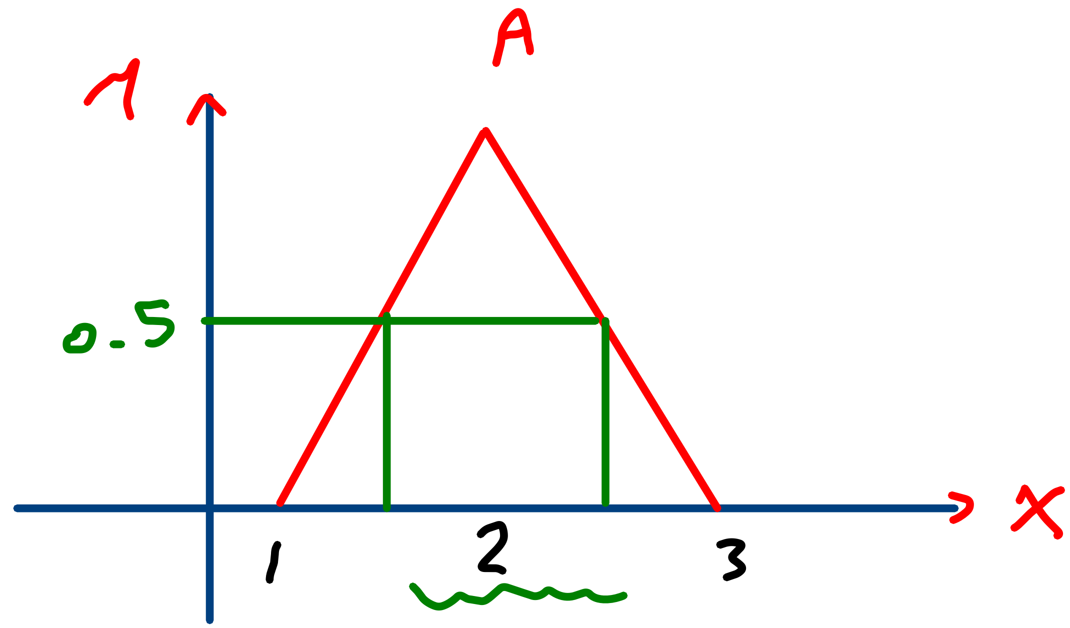
$$\mu_{\bigcup_{\alpha\in[0,1]} A^\alpha}(x) = \sup_{\alpha\in[0,\beta]} \alpha = \beta = \mu_A(x)$$

since the same argument it is valid for each $x \in X$, Then the proof is complete.

Example:- let A be a fuzzy set defined on $\mathbb{R}$ where

$$\mu_A(x) = \begin{cases} x-1 ; & x\in[1,2] \\ 3-x ; & x\in[2,3] \\ 0 ; & \text{otherw} \end{cases}$$

, find A_α and A^α



$x \in X$

$$A_\alpha = \{ \underline{\underline{x}} : \mu_A(x) \geq \alpha \}$$

$\alpha \in [0, 1]$

$$\mu_A(x) \geq \alpha \Rightarrow x - 1 \geq \alpha \Rightarrow x \geq \alpha + 1$$

and $\mu_A(x) \geq \alpha \Rightarrow 3 - x \geq \alpha \Rightarrow x \leq 3 - \alpha$

$$A_\alpha = [\alpha + 1, 3 - \alpha]$$

$A_0 = [1, 3]$, $A_{0.5} = [1.5, 2.5]$

$A_1 = [2, 2] = 2$

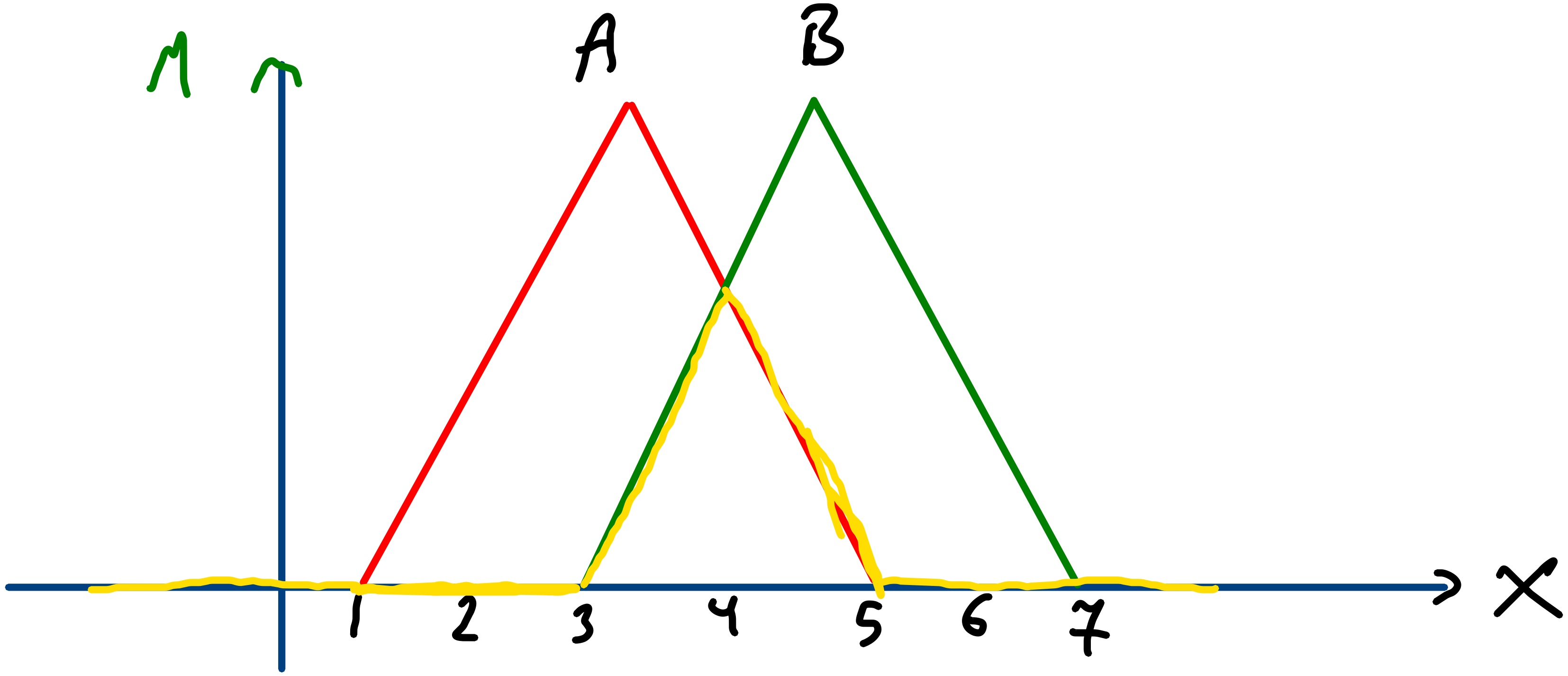
Intersection and Union of Continuous fuzzy set

$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$, $\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$

Example:- If A, B are fuzzy sets defined on the universal set $X = [0, 10]$ and whose memberships defined as follows

$$\mu_A(x) = \begin{cases} \frac{x-1}{2} & ; 1 \le x \le 3 \\ \frac{5-x}{2} & ; 3 \le x \le 5 \\ 0 & ; \text{otherwise} \end{cases} \qquad \mu_B(x) = \begin{cases} \frac{x-3}{2} & ; 3 \le x \le 5 \\ \frac{7-x}{2} & ; 5 \le x \le 7 \\ 0 & ; \text{otherwise} \end{cases}$$

find $A\cap B$, $A\cup B$



To find the intersection point of two curves, we do

$$\mu_A(x) = \mu_B(x) \Rightarrow \frac{5-x}{2} = \frac{x-3}{2} \Rightarrow x = 4$$

$$\mu_{A\cap B}(x) = \begin{cases} \mu_B(x) ; & 0 \le x \le 4 \\ \mu_A(x) ; & 4 \le x \le 10 \end{cases} = \left\{ \begin{array}{l} \overbrace{\begin{array}{ll} 0 ; & 0 \le x \le 3 \\ \frac{x-3}{2} ; & 3 \le x \le 4 \end{array}}^{\mu_B(x)} \\ \underbrace{\begin{array}{ll} \frac{5-x}{2} ; & 4 \le x \le 5 \\ 0 ; & 5 \le x \le 10 \end{array}}_{\mu_A(x)} \end{array} \right.$$