Theorem:- Let A_i be fuzzy sets on X $\forall$ $i \in I$, where I is an index of sets, Then

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① $\bigcup_{i\in I} (A_i)_\alpha \subseteq \left(\bigcup_{i \in I} A_i\right)_\alpha$ and $\bigcap_{i\in I} (A_i)_\alpha = \left(\bigcap_{i\in I} A_i\right)_\alpha$

② $\bigcup_{i\in I} (A_i)_{\alpha+} = \left(\bigcup_{i\in I} A_i\right)_{\alpha+}$ and $\bigcap_{i\in I} (A_i)_{\alpha+} \subseteq \left(\bigcap_{i\in I} A_i\right)_{\alpha+}$

Theorem :- let A, B be fuzzy sets on X, then for all $\alpha \in [0,1]$ then :- ① $A \subseteq B$ iff $A_\alpha \subseteq B_\alpha$ & $A \subseteq B$ iff $A_{\alpha+} \subseteq B_{\alpha+}$

② $A = B$ iff $A_\alpha = B_\alpha$ & $A = B$ iff $A_{\alpha+} = B_{\alpha+}$

proof ① ($A \subseteq B$ iff $A_\alpha \subseteq B_\alpha$), let $A \subseteq B \Rightarrow \mu_A(x) \leq \mu_B(x)$ $\forall x \in X$; let $x \in A_\alpha \Rightarrow \mu_A(x) \geq \alpha \Rightarrow \mu_B(x) \geq \alpha$
$\Rightarrow x \in B_\alpha \Rightarrow A_\alpha \subseteq B_\alpha$

Let $A_\alpha \subseteq B_\alpha \Rightarrow \forall\, x \in A_\alpha \Rightarrow x \in B_\alpha$

$\Rightarrow \mu_A(x) \geq \alpha$, $\mu_B(x) \geq \alpha \Rightarrow \exists\, x_1 \in B_\alpha$ and there is no exist in A_α

Let $x \in A \Rightarrow \mu_A(x) = \alpha$, $\Rightarrow \mu_A(x_1) \leq \mu_B(x_1)$

$\Rightarrow A \subseteq B$

$A = \{(0, 0.3), (4, 0.5), (2, 0.7), (5, 0.7)\}$

$\subseteq$

$A_{0.5} = \{4, 2, 5\}$

$\subseteq$

$B = \{(0, 0.3), (4, 0.7), (2, 0.8), (5, 0.9)$
$(6, 0.2)\}$

$B_{0.3} = \{4, 2, 5, 6\}$

Theorem:- let A be a fuzzy set on X then واجب

① $A_\alpha = \bigcap_{\beta<\alpha} A_\beta = \bigcap_{\beta<\alpha} A_{\beta+}$ ② $A_{\alpha+} = \bigcup_{\alpha<\beta} A_\beta = \bigcup_{\alpha<\beta} A_{\beta+}$

Representation of fuzzy sets

The principle role of α-cuts and strong α-cuts in fuzzy sets theory is their capability to represent the fuzzy set. To show that the representation of fuzzy set by α-cuts we will use the following example.

Let $X = \{ x_1, x_2, x_3, x_4, x_5 \}$

let $A = \{ (x_1, 0.2), (x_2, 0.4), (x_3, 0.6), (x_4, 0.8), (x_5, 1) \}$

then there exists only five distinict α-cuts set for A.

$$A_{0.2} = 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$A_{0.4} = 0/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$A_{0.6} = 0/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$A_{0.8} = 0/x_1 + 0/x_2 + 0/x_3 + 1/x_4 + 1/x_5$$

$$A_1 = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$

Now, we will convert each α-cuts to a special fuzzy sets A^{α} defined for each $x \in \underset{\sim}{X}$ as follow

$$A^{\alpha} = \alpha \cdot A_{\alpha} \Rightarrow \mu_{A^{\alpha}}(x) = \alpha \cdot \mu_{A_{\alpha}}(x)$$

$$A^{0.2} = 0.2/x_1 + 0.2/x_2 + 0.2/x_3 + 0.2/x_4 + 0.2/x_5$$

$$A^{0.4} = 0/x_1 + 0.4/x_2 + 0.4/x_3 + 0.4/x_4 + 0.4/x_5$$

$$A^{0.6} = 0/x_1 + 0/x_2 + 0.6/x_3 + 0.6/x_4 + 0.6/x_5$$

$$A^{0.8} = 0/x_1 + 0/x_2 + 0/x_3 + 0.8/x_4 + 0.8/x_5$$

$$A^{1} = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$

$$A = A^{0.2} \cup A^{0.4} \cup A^{0.6} \cup A^{0.8} \cup A^{1}$$