Tikrit University

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Multimedia Security

Watermark

DCT & IDCT Tutorial

Dr. Moceheb Lazam Shuwandy

Lecture: 8

-Discrete Cosine Transform (DCT)

$$C(u,v) = a(u)a(v) \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{\pi(2x+1)u}{2N}\right]\cos\left[\frac{\pi(2y+1)v}{2N}\right]$$

$$for\ u,v = 0,1,2,\ldots..N-1$$

$$\alpha(u) = \begin{cases}\sqrt{\dfrac{1}{N}} & for\ u=0 \\[2mm] \sqrt{\dfrac{2}{N}} & for\ u\neq 0\end{cases}$$

$$\alpha(v) = \begin{cases}\sqrt{\dfrac{1}{N}} & for\ v=0 \\[2mm] \sqrt{\dfrac{2}{N}} & for\ v\neq 0\end{cases}$$

For Example:

Get the DCT for the block of 2x2 image below:

2	3
4	1

Sol.:

N=2 → u and v=0,1

$$C(0,0) = \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2}} \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)$$

$$C(0,0) = \frac{1}{2} \times \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y) = 0.5 \times (2+3+4+1)$$

$$C(0,0) = 5$$

$$C(0,1) = \sqrt{\frac{1}{2}} \times \sqrt{\frac{2}{2}} \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y) \cos\left[\frac{\pi(2y+1)}{4}\right]$$

$$C(0,1) = 0.7071 \times [(2 \times 0.7071) + (3 \times -0.7071) + (4 \times 0.7071) + (1 \times -0.7071)]$$

$$C(0,1) = 0.7071 \times [1.414 - 2.121 + 2.828 - 0.7071]$$

$$C(0,1) = 0.9997 \cong 1$$

$$C(1,0) = \sqrt{\frac{2}{2}} \times \sqrt{\frac{1}{2}} \sum_{x=0}^{1} \sum_{y=0}^{1} f(x,y) \cos\left[\frac{\pi(2x+1)}{4}\right]$$

$$C(1,0) = 0.7071 \times [(2 \times 0.7071) + (3 \times 0.7071)] + [(4 \times -0.7071) + (1 \times -0.7071)]$$

$$C(1,0) = 0.7071 \times [1.4154 + 2.1231 - 2.828 - 0.7071]$$

$$C(1,0) = 0.7071 \times 0.0034 = 0.0024 \cong 0$$

$$C(1,1) = \sqrt{\frac{2}{2}} \times \sqrt{\frac{2}{2}} \sum_{x=0}^{1} \sum_{y=0}^{1} f(x,y) \cos\left[\frac{\pi(2x+1)}{4}\right] \cos\left[\frac{\pi(2y+1)}{4}\right]$$

$$C(1,1) = 1 \times [(2 \times 0.7071 \times 0.7071) + (3 \times 0.7071 \times -0.7071)] + [(4 \times -0.7071 \times 0.7071) + (1 \times -0.7071 \times -0.7071)]$$

$$C(1,1) = [0.99998082 - 1.49997123 - 1.99996164 + 0.49999041]$$

$$C(1,1) = -1.99996164 \cong -2$$

The result of DCT:

5	1
0	-2

-Inverse Discrete Cosine Transform (IDCT)

$$f(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} a(u)a(v) \, C(x,y) \cos\left[\frac{\pi(2x+1)u}{2N}\right] \cos\left[\frac{\pi(2y+1)v}{2N}\right]$$

$$for \; x,y = 0,1,2,\ldots\ldots N-1$$

$$\alpha(u) = \begin{cases} \sqrt{\dfrac{1}{N}} & for \; u = 0 \\[2mm] \sqrt{\dfrac{2}{N}} & for \; u \neq 0 \end{cases}$$

$$\alpha(v) = \begin{cases} \sqrt{\dfrac{1}{N}} & for \; v = 0 \\[2mm] \sqrt{\dfrac{2}{N}} & for \; v \neq 0 \end{cases}$$

For Example:

Get the IDCT for the block of 2x2 image below:

5	1
0	-2

Sol.:

N=2 → u and v=0,1

$$f(0,0) = \sum_{x=0}^{1}\sum_{y=0}^{1}\sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2}} \times C(x,y)$$

$$f(0,0) = \sum_{x=0}^{1}\sum_{y=0}^{1}\frac{1}{2} \times f(x,y) = 0.5 \times (5 + 1 + 0 - 2)$$

$$f(0,0) = 2$$

$$f(0,1) = \sum_{x=0}^{1}\sum_{y=0}^{1}\sqrt{\frac{1}{2}} \times \sqrt{\frac{2}{2}} \times C(x,y)\cos\left[\frac{\pi(2y+1)}{4}\right]$$

$$f(0,1) = 0.7071 \times [(5 \times 0.7071) + (1 \times -0.7071) + (0 \times 0.7071) + (-2 \times -0.7071)]$$

$$f(0,1) = 0.7071 \times [3.5355 - 0.7071 + 0 + 1.4142]$$

$$f(0,1) = 2.99994246 \cong 3$$

$$f(1,0) = \sum_{x=0}^{1}\sum_{y=0}^{1}\sqrt{\frac{2}{2}} \times \sqrt{\frac{1}{2}} \times C(x,y)\cos\left[\frac{\pi(2x+1)}{4}\right]$$

$$f(1,0) = 0.7071 \times [(5 \times 0.7071) + (1 \times 0.7071)] + [(0 \times -0.7071) + (-2 \times -0.7071)]$$

$$f(1,0) = 0.7071 \times [3.5355 + 0.7071 + 0 + 1.4142]$$

$$f(1,0) = 0.7071 \times 5.6568 = 3.99992328 \cong 4$$

$$f(1,1) = \sum_{x=0}^{1}\sum_{y=0}^{1}\sqrt{\frac{2}{2}} \times \sqrt{\frac{2}{2}} \times C(x,y)\cos\left[\frac{\pi(2x+1)}{4}\right]\cos\left[\frac{\pi(2y+1)}{4}\right]$$

$$f(1,1) = 1 \times [(5 \times 0.7071 \times 0.7071) + (1 \times 0.7071 \times -0.7071)] + [(0 \times -0.7071 \times 0.7071) + (-2 \times -0.7071 \times -0.7071)]$$

$$f(1,1) = [2.49995205 - 0.49999041 + 0 - 0.99998082]$$

$$f(1,1) = 0.99998082 \cong 1$$

The result of DCT:

2	3
4	1

For validating the example above by Octave-online:


```
          Vars
# ans
[2x2] cv
[2x2] uv
[2x2] xy
[2x2] xy2
```

```
octave:13> xy=[2 3; 4 1]
xy =

   2   3
   4   1

octave:14> uv=dct2(xy)
uv =

   5.00000   1.00000
   0.00000  -2.00000

octave:15> xy2=idct2(uv)
xy2 =

   2.0000   3.0000
   4.0000   1.0000

》|
```