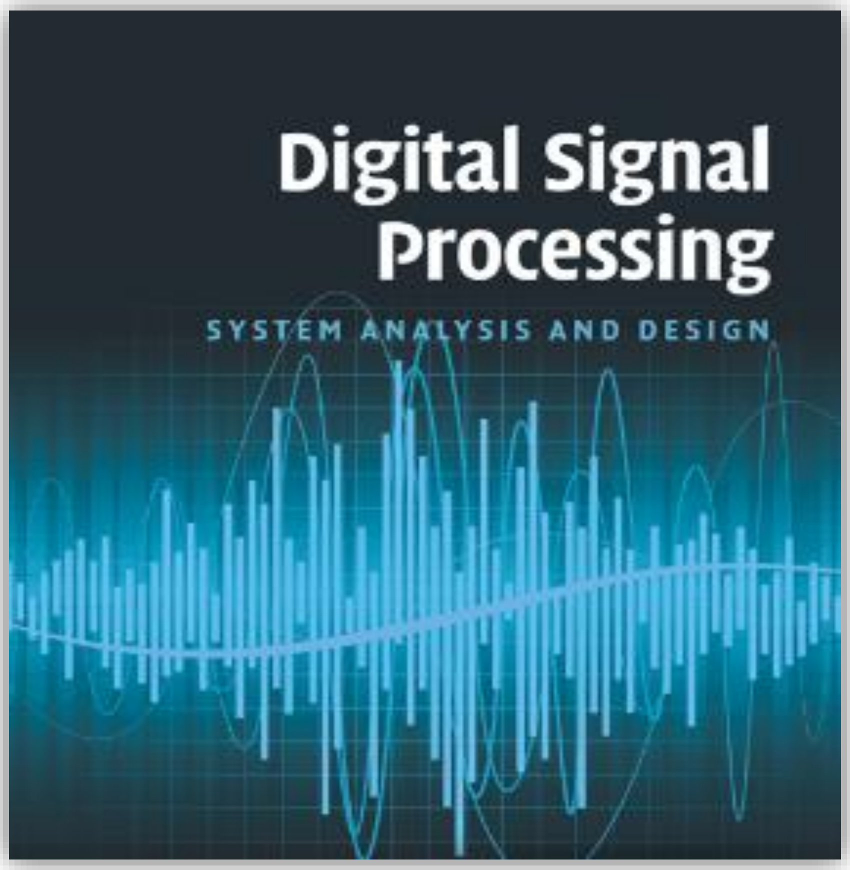

Tikrit University

Computer Science Dept.

Third Class

Lecture 8

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EXAMPLE 3 Find the convolution of the signals

$$x(n) = \begin{cases} 2 & n = -2, 0, 1 \\ 3 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \delta(n) - 2\delta(n-1) + 3\delta(n-2) - \delta(n-3)$$

Solution: Given $x(n) = \{2, 3, \underset{\uparrow}{2}, 2\}$; $h(n) = \{1, -2, 3, -1\}$

$x(n)$ starts at $n_1 = -2$ and $h(n)$ starts at $n_2 = 0$. The starting sample of $y(n)$ is at $n = n_1 + n_2 = -2 + 0 = -2$. Since number of samples in $x(n)$ is 4, and the number of samples in $h(n)$ is 4, the number of samples in $y(n)$ will be $4 + 4 - 1 = 7$. So $y(n)$ exists from -2 to 4.

Method 1 Graphical method

We know that $$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

From Figure 1 , we get

For $n = -2$ $$y(-2) = \sum_{k=-\infty}^{\infty} x(k)h(-2-k) = 2\cdot1 = 2$$

For $n = -1$ $$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k) = 2(-2) + 3\cdot1 = -1$$

For $n = 0$ $$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = 2\cdot3 + 3(-2) + 2\cdot1 = 2$$

For $n = 1$ $$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = 2(-1) + 3(3) + 2(-2) + 2\cdot1 = 5$$

For $n = 2$ $$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = 3(-1) + 2\cdot3 + 2(-2) = -1$$

For $n = 3$ $$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = 2(-1) + 2\cdot3 = 4$$

For $n = 4$ $$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = 2(-1) = -2$$

$$\therefore \quad y(n) = \{2, -1, \underset{\uparrow}{2}, 5, -1, 4, -2\}$$

To check the correctness of the result sum all the samples in $x(n)$ and multiply with the sum of all samples in $h(n)$. This value must be equal to sum of all samples in $y(n)$.

In the given problem, $\sum_n x(n) = 9$, $\sum_n h(n) = 1$, and $\sum_n y(n) = 9$.

This shows $\sum_n x(n) \cdot \sum_n h(n) = \sum_n y(n)$ (proved).

Therefore, the result is correct.

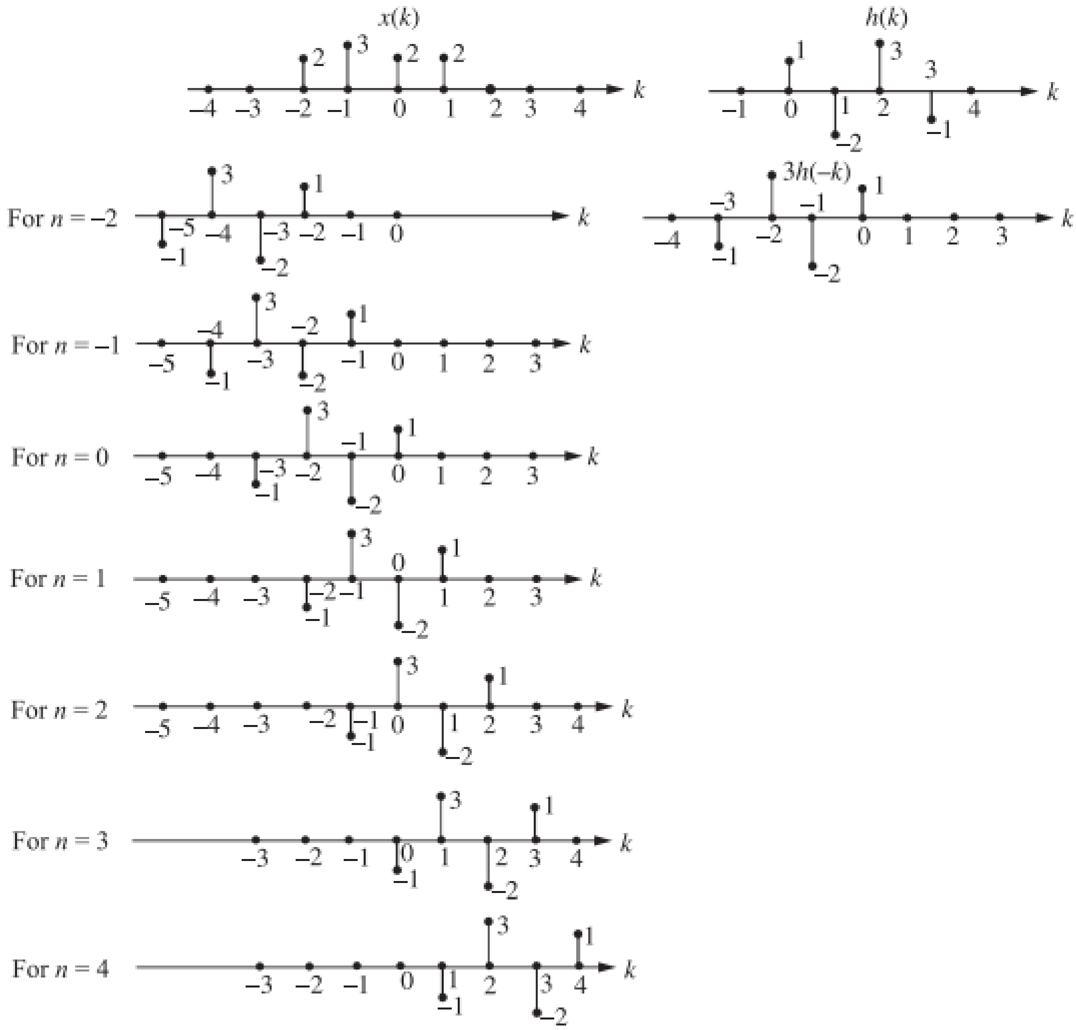

Figure 1 Operation on signals $x(n)$ and $h(n)$ to compute convolution.

Method 2 Tabular array

Tabulate the sequence $x(k)$ and shifted version of $h(k)$ as shown in Table **1**

Table **1** Table for computing $y(n)$.

k		−5	−4	−3	−2	−1	0	1	2	3	4	5	6
$x(k)$		–	–	–	2	3	2	2	–	–	–	–	–
$h(-k)$		–	–	−1	3	−2	1	–	–	–	–	–	–
$n = -2$	$h(-2-k)$	−1	3	−2	1	–	–	–	–	–	–	–	–
$n = -1$	$h(-1-k)$	–	−1	3	−2	1	–	–	–	–	–	–	–
$n = 0$	$h(-k)$	–	–	−1	3	−2	1	–	–	–	–	–	–
$n = 1$	$h(1-k)$	–	–	–	−1	3	−2	1	–	–	–	–	–
$n = 2$	$h(2-k)$	–	–	–	–	−1	3	−2	1	–	–	–	–
$n = 3$	$h(3-k)$	–	–	–	–	–	−1	3	−2	1	–	–	–
$n = 4$	$h(4-k)$	–	–	–	–	–	–	−1	3	−2	1	–	–

The starting sample of $y(n)$ is at $n = -2$. $y(n)$ is calculated as shown below. From the table, we can see that

For $n = -2$ $\quad y(-2) = \sum_{k=-\infty}^{\infty} x(k)h(-2-k) = 2 \cdot 1 = 2$

For $n = -1$ $\quad y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k) = 2(-2) + 3 \cdot 1 = -1$

For $n = 0$ $\quad y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = 2 \cdot 3 + 3(-2) + 2 \cdot 1 = 2$

For $n = 1$ $\quad y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = 2(-1) + 3(3) + 2(-2) + 2 \cdot 1 = 5$

For $n = 2$ $\quad y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = 3(-1) + 2 \cdot 3 + 2(-2) = -1$

For $n = 3$ $\quad y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = 2(-1) + 2 \cdot 3 = 4$

For $n = 4$ $\quad y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = 2(-1) = -2$

$$\therefore \quad y(n) = \left\{ \begin{matrix} 2, -1, 2, 5, -1, 4, -2 \\ \uparrow \end{matrix} \right\}$$

Method 3 *Tabular method*

$y(n) = x(n) * h(n)$ is computed by tabular method as shown in Table 2.

TABLE 2. Table for computing $y(n)$.

$h(n)$ \ $x(n)$	2	3	2	2
1	2	3	2	2
−2	−4	−6	−4	−4
3	6	9	6	6
−1	−2	−3	−2	−2

$$\therefore \quad y(n) = 2, -4 + 3, 6 - 6 + 2, -2 + 9 - 4 + 2, -3 + 6 - 4, -2 + 6, -2$$

$$= \left\{ \begin{matrix} 2, -1, 2, 5, -1, 4, -2 \\ \uparrow \end{matrix} \right\}$$

Method 4 Matrices method

The given sequences are: $x(n) = \{x(0), x(1), x(2), x(3)\} = \{2, \underset{\uparrow}{3}, 2, 2\}$

and $h(n) = \{h(0), h(1), h(2), h(3)\} = \{1, \underset{\uparrow}{-2}, 3, -1\}$

The sequence $x(n)$ is starting at $n = -1$ and the sequence $h(n)$ is also starting at $n = -1$. So the sequence $y(n)$ corresponding to the linear convolution of $x(n)$ and $y(n)$ will start at $n = -1 + (-1) = -2$. $x(n)$ is of length 4 and $h(n)$ is also of length 4. So length of $y(n) = 4 + 4 - 1 = 7$. Substituting the sequence values in matrix form and multiplying, we get the convolution of $x(n)$ and $h(n)$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ -1 & 3 & -2 & 1 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \\ 5 \\ -1 \\ 4 \\ -2 \end{bmatrix}$$