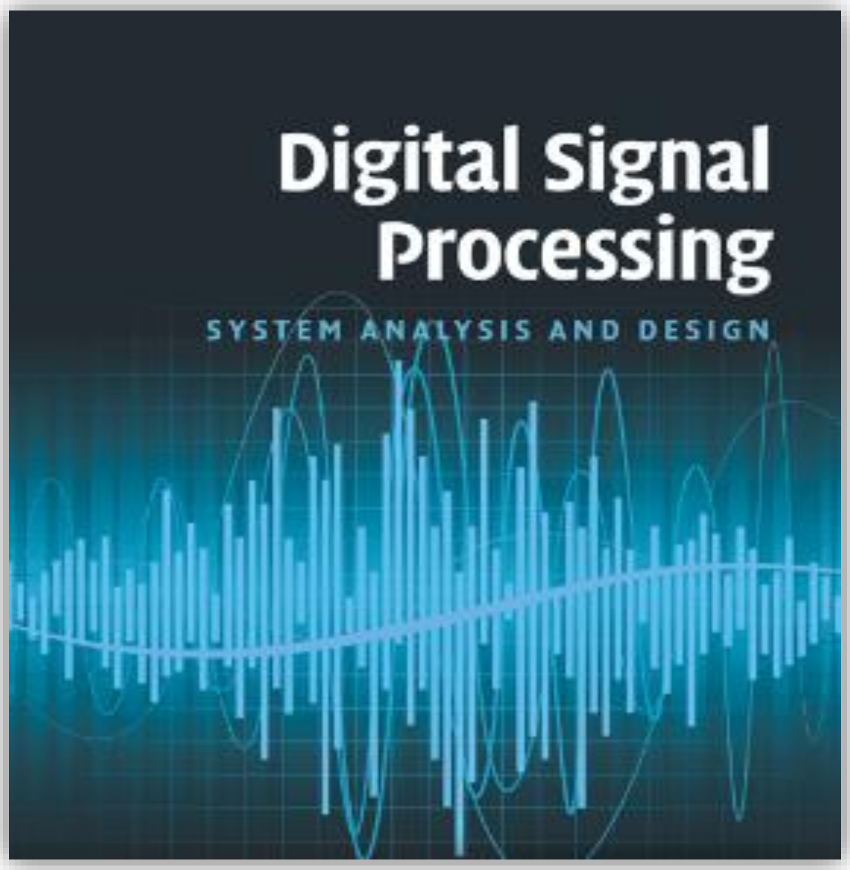

Tikrit University

Computer Science Dept.

Third Class

Lecture 6

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4.5 Even and Odd Signals

Any signal $x(n)$ can be expressed as sum of even and odd components. That is

$$x(n) = x_e(n) + x_o(n)$$

where $x_e(n)$ is even components and $x_o(n)$ is odd components of the signal.

Even (symmetric) signal

A discrete-time signal $x(n)$ is said to be an even (symmetric) signal if it satisfies the condition:

$$x(n) = x(-n) \quad \text{for all } n$$

Even signals are symmetrical about the vertical axis or time origin. Hence they are also called symmetric signals: cosine sequence is an example of an even signal. Some even signals are shown in Figure 1. (a). An even signal is identical to its reflection about the origin. For an even signal $x_0(n) = 0$.

Odd (anti-symmetric) signal

A discrete-time signal $x(n)$ is said to be an odd (anti-symmetric) signal if it satisfies the condition:

$$x(-n) = -x(n) \quad \text{for all } n$$

Odd signals are anti-symmetrical about the vertical axis. Hence they are called anti-symmetric signals. Sinusoidal sequence is an example of an odd signal. For an odd signal $x_e(n) = 0$. Some odd signals are shown in Figure 1.(b).

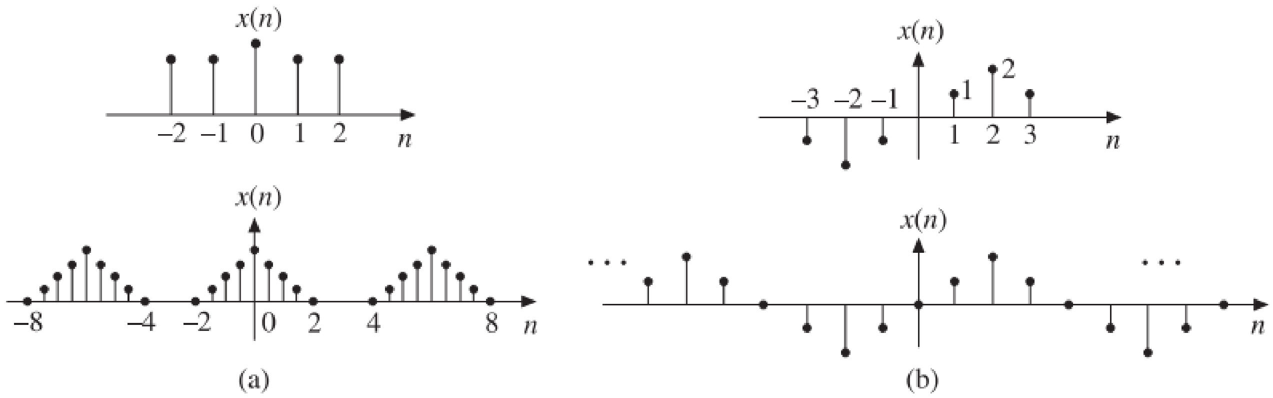

Figure 1 (a) Even sequences (b) Odd sequences.

Evaluation of even and odd parts of a signal

We have $$x(n) = x_e(n) + x_o(n)$$

$\therefore$ $$x(-n) = x_e(-n) + x_o(-n) = x_e(n) - x_o(n)$$

$$x(n) + x(-n) = x_e(n) + x_o(n) + x_e(n) - x_o(n) = 2x_e(n)$$

$\therefore$ $$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$x(n) - x(-n) = [x_e(n) + x_o(n)] - [x_e(n) - x_o(n)] = 2x_o(n)$$

$\therefore$ $$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

The product of two even or odd signals is an even signal and the product of even signal and odd signal is an odd signal.

We can prove this as follows:

Let $$x(n) = x_1(n)\, x_2(n)$$

(a) If $x_1(n)$ and $x_2(n)$ are both even, i.e.

$$x_1(-n) = x_1(n)$$

and $$x_2(-n) = x_2(n)$$

Then $$x(-n) = x_1(-n)x_2(-n) = x_1(n)x_2(n) = x(n)$$

Therefore, $x(n)$ is an even signal.

If $x_1(n)$ and $x_2(n)$ are both odd, i.e.

$$x_1(-n) = -x_1(n)$$

and $$x_2(-n) = -x_2(n)$$

Then $$x(-n) = x_1(-n)x_2(-n) = [-x_1(n)][-x_2(n)] = x_1(n)x_2(n) = x(n)$$

Therefore, $x(n)$ is an even signal.

(b) If $x_1(n)$ is even and $x_2(n)$ is odd, i.e.

$$x_1(-n) = x_1(n)$$

and $$x_2(-n) = -x_2(n)$$

Then $$x(-n) = x_1(-n)x_2(-n) = -x_1(n)x_2(n) = -x(n)$$

Therefore, $x(n)$ is an odd signal.

Thus, the product of two even signals or of two odd signals is an even signal, and the product of even and odd signals is an odd signal.

Every signal need not be either purely even signal or purely odd signal, but every signal can be decomposed into sum of even and odd parts.

Example 7 Find the even and odd components of the following signals:

(a) $x(n) = \{-3, 1, \underset{\uparrow}{2}, -4, 2\}$

(b) $x(n) = \{-2, 5, \underset{\uparrow}{1}, -3\}$

(c) $x(n) = \{\underset{\uparrow}{5}, 4, 3, 2, 1\}$

(d) $x(n) = \{5, 4, 3, 2, \underset{\uparrow}{1}\}$

Solution:

(a) Given $$x(n) = \{-3, 1, \underset{\uparrow}{2}, -4, 2\}$$

$$\therefore \quad x(-n) = \{2, -4, \underset{\uparrow}{2}, 1, -3\}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}\left[-3+2, 1-4, 2+2, -4+1, 2-3\right]$$

$$= \left\{-0.5, -1.5, \underset{\uparrow}{2}, -1.5, -0.5\right\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}\left[-3-2, 1+4, 2-2, -4-1, 2+3\right]$$

$$= \left\{-2.5, 2.5, \underset{\uparrow}{0}, -2.5, 2.5\right\}$$

(b) Given $x(n) = \left\{-2, 5, \underset{\uparrow}{1}, -3\right\}$

$$x(-n) = \left\{-3, \underset{\uparrow}{1}, 5, -2\right\}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}\left[-2+0, 5-3, 1+1, -3+5, 0-2\right]$$

$$= \left\{-1, 1, \underset{\uparrow}{1}, 1, -1\right\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}\left[-2-0, 5+3, 1-1, -3-5, 0+2\right]$$

$$= \left\{-1, 4, \underset{\uparrow}{0}, -4, 1\right\}$$

(c) Given $x(n) = \left\{\underset{\uparrow}{5}, 4, 3, 2, 1\right\}$

$$n = 0, 1, 2, 3, 4$$

$\therefore$ $x(n) = \underset{\uparrow}{5}, 4, 3, 2, 1$

$$x(-n) = 1, 2, 3, 4, \underset{\uparrow}{5}$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}[1, 2, 3, 4, 5 \underset{\uparrow}{+} 5, 4, 3, 2, 1]$$

$$= \left\{0.5, 1, 1.5, 2, \underset{\uparrow}{5}, 2, 1.5, 1, 0.5\right\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[-1, -2, -3, -4, 5 \underset{\uparrow}{-} 5, 4, 3, 2, 1]$$

$$= \left\{-0.5, -1, -1.5, -2, \underset{\uparrow}{0}, 2, 1.5, 1, 0.5\right\}$$

(d) Given $x(n) = \left\{5, 4, 3, 2, \underset{\uparrow}{1}\right\}$

$$n = -4, -3, -2, -1, 0$$

$\therefore$ $x(n) = 5, 4, 3, 2, \underset{\uparrow}{1}$

$$x(-n) = \underset{\uparrow}{1}, 2, 3, 4, 5$$

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}[5, 4, 3, 2, 1 \underset{\uparrow}{+} 1, 2, 3, 4, 5]$$

$$= [2.5, 2, 1.5, 1, \underset{\uparrow}{1}, 1, 1.5, 2, 2.5]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[5, 4, 3, 2, 1 \underset{\uparrow}{-} 1, -2, -3, -4, -5]$$

$$= \frac{1}{2}[2.5, 2, 1.5, 1, \underset{\uparrow}{0}, -1, -1.5, -2, -2.5]$$

When the signal is given as a waveform

The even part of the signal can be found by folding the signal about the y-axis and adding the folded signal to the original signal and dividing the sum by two. The odd part of the signal can be found by folding the signal about y-axis and subtracting the folded signal from the original signal and dividing the difference by two as illustrated in Figure 2.

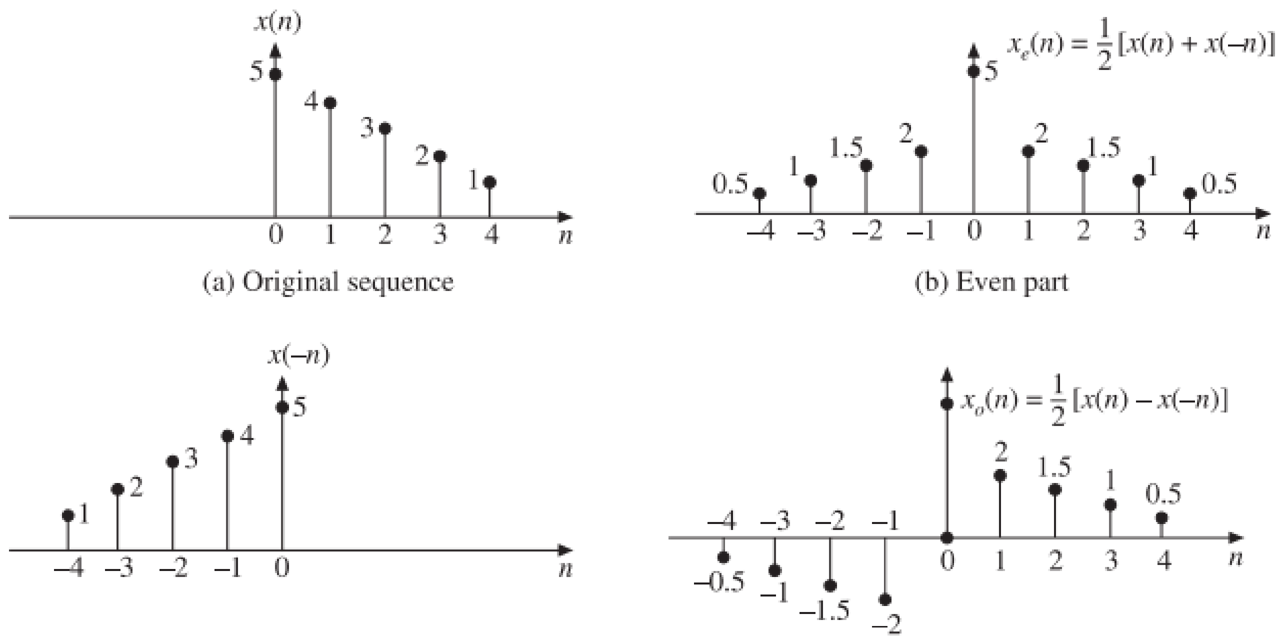

Figure 2 Graphical evaluation of even and odd parts.