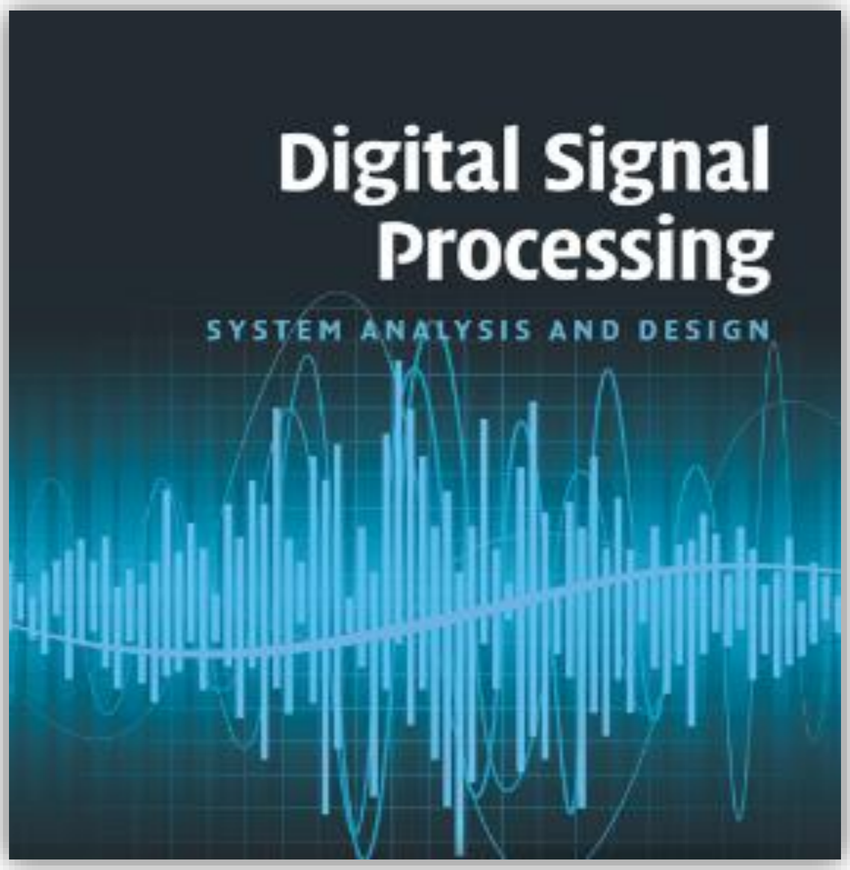

Tikrit University

Computer Science Dept.

Third Class

Lecture 5

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4.4 Energy and Power Signals

Signals may also be classified as energy signals and power signals. However there are some signals which can neither be classified as energy signals nor power signals.

The total energy E of a discrete-time signal $x(n)$ is defined as:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

and the average power P of a discrete-time signal $x(n)$ is defined as:

$$P = \underset{N\to\infty}{\mathrm{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

or $P = \frac{1}{N}\sum_{n=0}^{n-1} |x(n)|^2$ for a digital signal with $x(n) = 0$ for $n < 0$.

A signal is said to be an energy signal if and only if its total energy E over the interval $(-\infty, \infty)$ is finite (i.e., $0 < E < \infty$). For an energy signal, average power $P = 0$. Non-periodic signals which are defined over a finite time (also called time limited signals) are the examples of energy signals. Since the energy of a periodic signal is always either zero or infinite, any periodic signal cannot be an energy signal.

A signal is said to be a power signal, if its average power P is finite (i.e., $0 < P < \infty$). For a power signal, total energy $E = \infty$. Periodic signals are the examples of power signals. Every bounded and periodic signal is a power signal. But it is true that a power signal is not necessarily a bounded and periodic signal.

Both energy and power signals are mutually exclusive, i.e. no signal can be both energy signal and power signal.

The signals that do not satisfy the above properties are neither energy signals nor power signals. For example, $x(n) = u(n)$, $x(n) = nu(n)$, $x(n) = n^2u(n)$.

These are signals for which neither P nor E are finite. If the signals contain infinite energy and zero power or infinite energy and infinite power, they are neither energy nor power signals.

If the signal amplitude becomes zero as $|n| \to \infty$, it is an energy signal, and if the signal amplitude does not become zero as $|n| \to \infty$, it is a power signal.

Example 5 Find which of the following signals are energy signals, power signals, neither energy nor power signals:

(a) $\left(\frac{1}{2}\right)^n u(n)$ (b) $e^{j[(\pi/3)n+(\pi/2)]}$

(c) $\sin\left(\frac{\pi}{3}n\right)$ (d) $u(n) - u(n-6)$

(e) $nu(n)$ (f) $r(n) - r(n-4)$

Solution:

(a) Given $$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

Energy of the signal $E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} |x(n)|^2$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \left[\left(\frac{1}{2}\right)^n\right]^2 u(n)$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-(1/4)} = \frac{4}{3} \text{ joule}$$

Power of the signal $P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \left[\frac{1-(1/4)^{N+1}}{1-(1/4)}\right]$$

$$= 0$$

The energy is finite and power is zero. Therefore, $x(n)$ is an energy signal.

(b) Given $$x(n) = e^{j[(\pi/3)n + (\pi/2)]}$$

Energy of the signal $E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \left|e^{j[(\pi/3)n + (\pi/2)]}\right|^2$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} 1$$

$$= \underset{N\to\infty}{\text{Lt}} [2N+1] = \infty$$

Power of the signal $P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|e^{j[(\pi/3)n + (\pi/2)]}\right|^2$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} [2N+1] = 1 \text{ watt}$$

The energy is infinite and the power is finite. Therefore, it is a power signal.

(c) Given $$x(n) = \sin\left(\frac{\pi}{3} n\right)$$

Energy of the signal $$E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{3} n\right)$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \frac{1-\cos[(2\pi/3)\, n]}{2}$$

$$= \frac{1}{2} \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} \left(1 - \cos\frac{2\pi}{3} n\right)$$

$$= \infty$$

Power of the signal $$P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{3} n\right)$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos[(2\pi/3)\, n]}{2}$$

$$= \frac{1}{2} \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} [2N+1] = \frac{1}{2} \text{ watt}$$

The energy is infinite and power is finite. Therefore, it is a power signal.

(d) Given $$x(n) = u(n) - u(n-6)$$

Energy of the signal $$E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} [u(n) - u(n-6)]^2$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=0}^{5} 1 = 6 \text{ joule}$$

Power of the signal $$P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} [u(n) - u(n-6)]^2$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=0}^{5} 1 = 0$$

Energy is finite and power is zero. Therefore, it is an energy signal.

(e) Given $$x(n) = nu(n)$$

Energy of the signal $$E = \underset{N\to\infty}{\text{Lt}} \sum_{n=-N}^{N} [n]^2 u(n)$$

$$= \underset{N\to\infty}{\text{Lt}} \sum_{n=0}^{N} [n^2]$$

$$= \infty$$

Power of the signal $P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} [n]^2 u(n)$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

$$= \infty$$

Energy is infinite and power is also infinite. Therefore, it is neither energy signal nor power signal.

Example 6 Find whether the signal

$$x(n) = \begin{cases} n^2 & 0 \le n \le 3 \\ 10-n & 4 \le n \le 6 \\ n & 7 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

is a power signal or an energy signal. Also find the energy and power of the signal.

Solution: The given signal is a non-periodic finite duration signal. So it has finite energy and zero average power. So it is an energy signal.

Energy of the signal $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} (n)^2$$

$$= \sum_{n=0}^{3} n^4 + \sum_{n=4}^{6} (100 + n^2 - 20n) + \sum_{n=7}^{9} n^2$$

$$= (0+1+16+81) + (36+25+16) + (49+64+81)$$

$$= 369 < \infty \text{ joule}$$

Power of the signal $P = \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} \left[\sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} (n)^2 \right]$$

$$= \underset{N\to\infty}{\text{Lt}} \frac{1}{2N+1} [369] = 0$$

Here energy is finite and power is zero. So it is an energy signal.