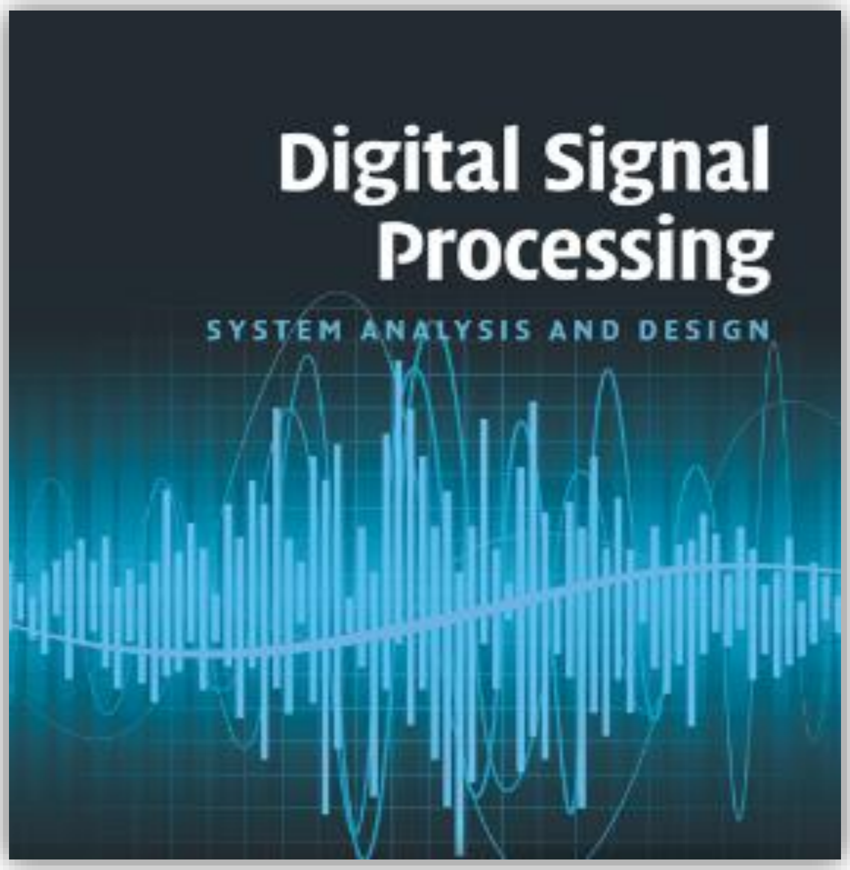

Tikrit University

Computer Science Dept.

Third Class

Lecture 4

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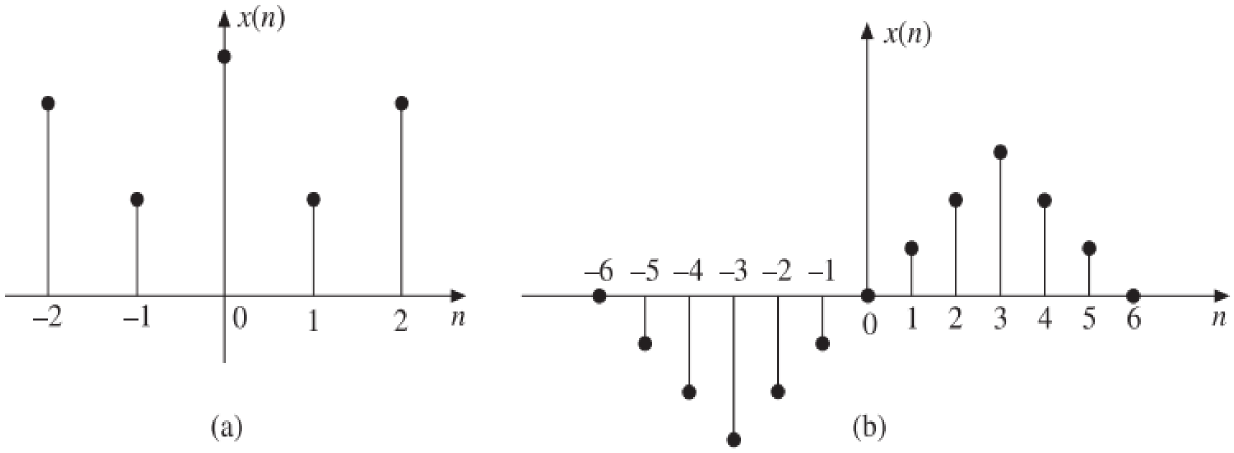
4.1 Classification of Signals

The signals can be classified based on their nature and characteristics in the time domain. They are broadly classified as: (i) continuous-time signals and (ii) discrete-time signals. The signals that are defined for every instant of time are known as continuous-time signals. The continuous-time signals are also called analog signals. They are denoted by x(t). They are continuous in amplitude as well as in time. Most of the signals available are continuous-time signals. The signals that are defined only at discrete instants of time are known as discrete-time signals. The discrete-time signals are continuous in amplitude, but discrete in time. For discrete-time signals, the amplitude between two-time instants is just not defined. For discrete-time signals, the independent variable is time n. Since they are defined only at discrete instants of time, they are denoted by a sequence x(nT) or simply by x(n) where n is an integer. Figure 1(a, b) shows the graphical representation of discrete-time signals.



Both continuous-time and discrete-time signals are further classified as follows:

1. Deterministic and random signals

2. Periodic and Aperiodic signals

3. Causal and non-causal signals.

4. Energy and power signals

5. Even and odd signals

4.1.1 Deterministic and Random Signals

A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called deterministic signal. A deterministic signal can be completely represented by mathematical equation at any time and its nature and amplitude at any time can be predicted.

Examples: Sinusoidal sequence $x(n) = \cos \omega n$, Exponential sequence $x(n) = e^{j\omega n}$, ramp sequence $x(n) = \alpha n$.

A signal characterized by uncertainty about its occurrence is called a non-deterministic or random signal. A random signal cannot be represented by any mathematical equation. The behaviour of such a signal is probabilistic in nature and can be analyzed only stochastically. The pattern of such a signal is quite irregular. Its amplitude and phase at any time instant cannot be predicted in advance. A typical example of a non-deterministic signal is thermal noise.

4.2 Periodic and Aperiodic Signals

A signal which has a definite pattern and repeats itself at regular intervals of time is called a periodic signal, and a signal which does not repeat at regular intervals of time is called a non-periodic or aperiodic signal. A discrete-time signal $x(n)$ is said to be periodic if it satisfies the condition $x(n) = x(n + N)$ for all integers n. The smallest value of N which satisfies the above condition is known as fundamental period.

If the above condition is not satisfied even for one value of n, then the discrete-time signal is aperiodic. Sometimes aperiodic signals are said to have a period equal to infinity.

The angular frequency is given by

$$\omega = \frac{2\pi}{N}$$

∴ Fundamental period $$N = \frac{2\pi}{\omega}$$

The sum of two discrete-time periodic sequences is always periodic.

Some examples of discrete-time periodic/non-periodic signals are shown in **Figure 2**

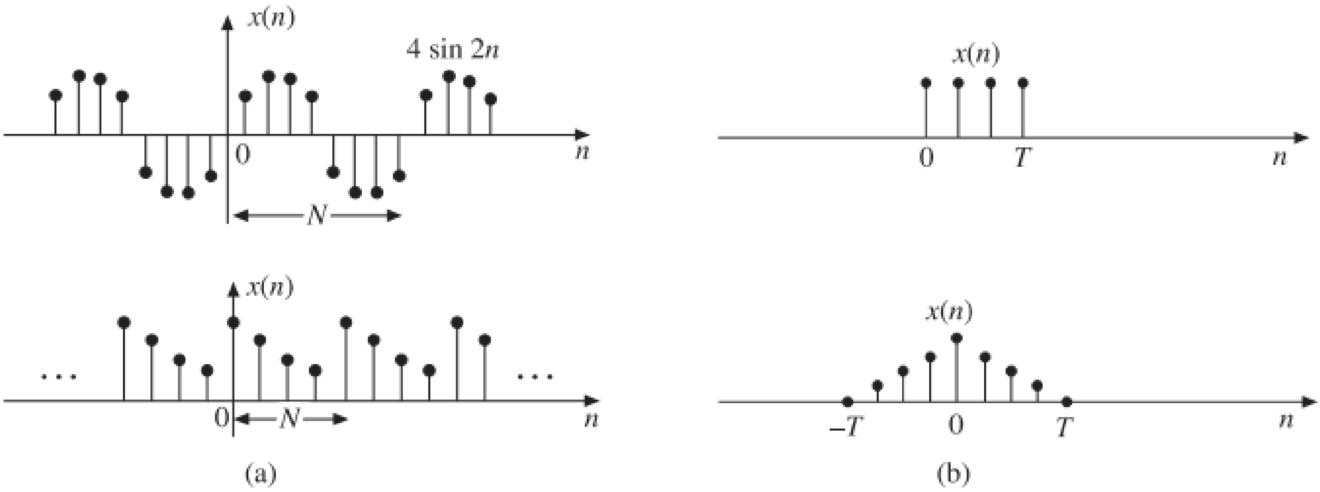


Figure 2 Examples of discrete-time: (a) Periodic and (b) Non-periodic signals.

Example 1 Show that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic only if $\omega_0/2\pi$ is a rational number.

Solution: Given $$x(n) = e^{j\omega_0 n}$$

$x(n)$ will be periodic if $$x(n + N) = x(n)$$

i.e. $$e^{j[\omega_0(n+N)]} = e^{j\omega_0 n}$$

i.e. $$e^{j\omega_0 N} e^{j\omega_0 n} = e^{j\omega_0 n}$$

This is possible only if $$e^{j\omega_0 N} = 1$$

This is true only if $$\omega_0 N = 2\pi k$$

where k is an integer.

∴ $$\frac{\omega_0}{2\pi} = \frac{k}{N} \text{ Rational number}$$

This shows that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic if $\omega_0/2\pi$ is a rational number.

Example 2 Obtain the condition for discrete-time sinusoidal signal to be periodic.

Solution: In case of continuous-time signals, all sinusoidal signals are periodic. But in discrete-time case, not all sinusoidal sequences are periodic.

Consider a discrete-time signal given by

$$x(n) = A \sin(\omega_0 n + \theta)$$

where A is amplitude, ω_0 is frequency and θ is phase shift.

A discrete-time signal is periodic if and only if

$$x(n) = x(n + N) \text{ for all } n$$

Now, $$x(n + N) = A \sin[\omega_0(n + N) + \theta)] = A \sin(\omega_0 n + \theta + \omega_0 N)$$

Therefore, $x(n)$ and $x(n + N)$ are equal if $\omega_0 N = 2\pi m$. That is, there must be an integer m such that

$$\omega_0 = \frac{2\pi m}{N} = 2\pi\left[\frac{m}{N}\right]$$

or $$N = 2\pi\left[\frac{m}{\omega_0}\right]$$

From the above equation we find that, for the discrete-time signal to be periodic, the fundamental frequency ω_0 must be a rational multiple of 2π. Otherwise the discrete-time signal is aperiodic. The smallest value of positive integer N, for some integer m, is the fundamental period.

Example 3 Determine whether the following discrete-time signals are periodic or not. If periodic, determine the fundamental period.

(a) $\sin(0.02\pi n)$

(b) $\sin(5\pi n)$

(c) $\cos 4n$

(d) $\sin\frac{2\pi n}{3} + \cos\frac{2\pi n}{5}$

(e) $\cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$

(f) $\cos\left(\frac{\pi}{2} + 0.3n\right)$

(g) $e^{j(\pi/2)n}$

(h) $1 + e^{j2\pi n/3} - e^{j4\pi n/7}$

Solution:

(a) Given $x(n) = \sin(0.02\pi n)$

Comparing it with $x(n) = \sin(2\pi f n)$

we have $0.02\pi = 2\pi f$ or $f = \frac{0.02\pi}{2\pi} = 0.01 = \frac{1}{100} = \frac{k}{N}$

Here f is expressed as a ratio of two integers with $k = 1$ and $N = 100$. So it is rational. Hence the given signal is periodic with fundamental period $N = 100$.

(b) Given $x(n) = \sin(5\pi n)$

Comparing it with $x(n) = \sin(2\pi f n)$

we have $2\pi f = 5\pi$ or $f = \frac{5}{2} = \frac{k}{N}$

Here f is a ratio of two integers with $k = 5$ and $N = 2$. Hence it is rational. Hence the given signal is periodic with fundamental period $N = 2$.

(c) Given $x(n) = \cos 4n$

Comparing it with $x(n) = \cos 2\pi f n$

we have $2\pi f = 4$ or $f = \frac{2}{\pi}$

Since $f = (2/\pi)$ is not a rational number, $x(n)$ is not periodic.

(d) Given $x(n) = \sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$

Comparing it with $x(n) = \sin 2\pi f_1 n + \cos 2\pi f_2 n$

we have $2\pi f_1 = \frac{2\pi}{3}$ or $f_1 = \frac{1}{3} = \frac{k_1}{N_1}$

$\therefore \quad N_1 = 3$

and $2\pi f_2 n = \frac{2\pi}{5}$ or $f_2 = \frac{1}{5}$

$\therefore \quad N_2 = 5$

Since $\frac{N_1}{N_2} = \frac{3}{5}$ is a ratio of two integers, the sequence $x(n)$ is periodic. The period of $x(n)$ is the LCM of N_1 and N_2. Here LCM of $N_1 = 3$ and $N_2 = 5$ is 15. Therefore, the given sequence is periodic with fundamental period $N = 15$.

(e) Given $x(n) = \cos\left(\frac{n}{6}\right) \cos\left(\frac{n\pi}{6}\right)$

Comparing it with $x(n) = \cos(2\pi f_1 n) \cos(2\pi f_2 n)$

we have $2\pi f_1 n = \frac{n}{6}$ or $f_1 = \frac{1}{12\pi}$

which is not rational.

And $2\pi f_2 n = \frac{n\pi}{6}$ or $f_2 = \frac{1}{12}$

which is rational.

Thus, $\cos(n/6)$ is non-periodic and $\cos(n\pi/6)$ is periodic. $x(n)$ is non-periodic because it is the product of periodic and non-periodic signals.

(f) Given $$x(n) = \cos\left(\frac{\pi}{2} + 0.3n\right)$$

Comparing it with $$x(n) = \cos(2\pi fn + \theta)$$

we have $2\pi fn = 0.3n$ and phase shift $\theta = \frac{\pi}{2}$

$$\therefore \qquad f = \frac{0.3}{2\pi} = \frac{3}{20\pi}$$

which is not rational.
Hence, the signal $x(n)$ is non-periodic.

(g) Given $$x(n) = e^{j(\pi/2)n}$$

Comparing it with $$x(n) = e^{j2\pi fn}$$

we have $2\pi f = \frac{\pi}{2}$ or $f = \frac{1}{4} = \frac{k}{N}$

which is rational.
Hence, the given signal $x(n)$ is periodic with fundamental period $N = 4$.

4.3 Causal and Non-causal Signals

A discrete-time signal $x(n)$ is said to be causal if $x(n) = 0$ for $n < 0$, otherwise the signal is non-causal. A discrete-time signal $x(n)$ is said to be anti-causal if $x(n) = 0$ for $n > 0$.

A causal signal does not exist for negative time and an anti-causal signal does not exist for positive time. A signal which exists in positive as well as negative time is called a non-casual signal.

$u(n)$ is a causal signal and $u(-n)$ an anti-causal signal, whereas $x(n) = 1$ for $-2 \leq n \leq 3$ is a non-causal signal.

Example 4 Find which of the following signals are causal or non-causal.

(a) $x(n) = u(n+4) - u(n-2)$

(b) $x(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4)$

(c) $x(n) = u(-n)$

Solution:

(a) Given $x(n) = u(n+4) - u(n-2)$
The given signal exists from $n = -4$ to $n = 1$. Since $x(n) \neq 0$ for $n < 0$, it is non-causal.

(b) Given $x(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4)$

The given signal exists for $n < 0$ also. So it is non-causal.

(c) Given $x(n) = u(-n)$
The given signal exists only for $n < 0$. So it is anti-causal. It can be called non-causal also.