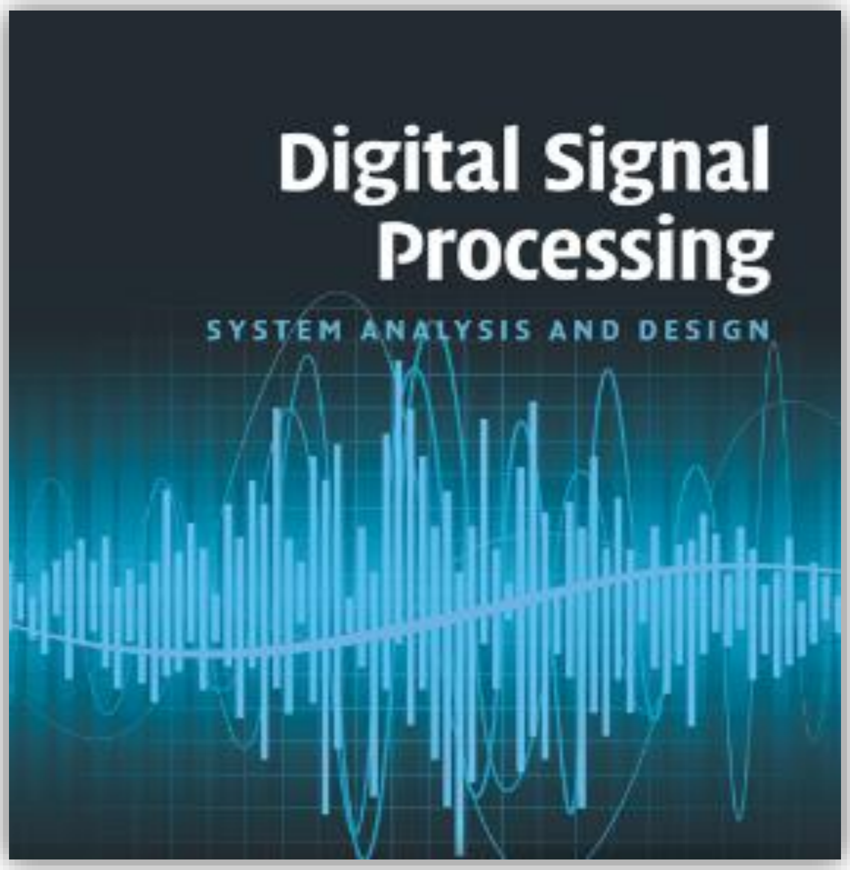

Tikrit University

Computer Science Dept.

Third Class

Lecture 2

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2. Elementary signals

There are several elementary signals which play vital role in the study of signal processing. These elementary signals serve as basic building blocks for the construction of more complex signals. In fact, these elementary signals may be used to model a large number of physical signals, which occur in nature. These elementary signals are also called standard signals.

The standard signals are as follows:

1. Unit step Function
2. Unit ramp Function
3. Unit parabolic Function
4. Unit impulse Function
5. Sinusoidal Function
6. Real exponential Function
7. Complex exponential Function

2.1 The Unit Step Function

The step Function is an important signal used for analysis of many continuous and discrete-time systems. It exists only for positive time and is zero for negative time. It is equivalent to applying a signal whose amplitude suddenly changes and remains constant at the sampling instants forever after application. In between the discrete instants it is zero. If a step function has unity magnitude, then it is called unit step function. The usefulness of the unit-step function lies in the fact that if we want a signal to start at $t = 0$, so that it may have a value of zero for $t < 0$, we only need to multiply the given signal with unit step function $u(t)$.

The continuous-time unit step function $u(t)$ is defined as:

$$u(t)=\begin{cases}1 & \text{for } t\geq 0\\ 0 & \text{for } t<0\end{cases}$$

From the above equation for $u(t)$, we can observe that when the argument t in $u(t)$ is less than zero, then the unit step function is zero, and when the argument t in $u(t)$ is greater than or equal to zero, then the unit step function is unity.

The shifted unit step function $u(t-a)$ is defined as:

$$u(t-a)=\begin{cases}1 & \text{for } t\geq a\\ 0 & \text{for } t<a\end{cases}$$

It is zero if the argument $(t-a)<0$ and equal to 1 if the argument $(t-a)\geq 0$.
The graphical representations of $u(t)$ and $u(t-a)$ are shown in Figure 1.[(a) and (b)].

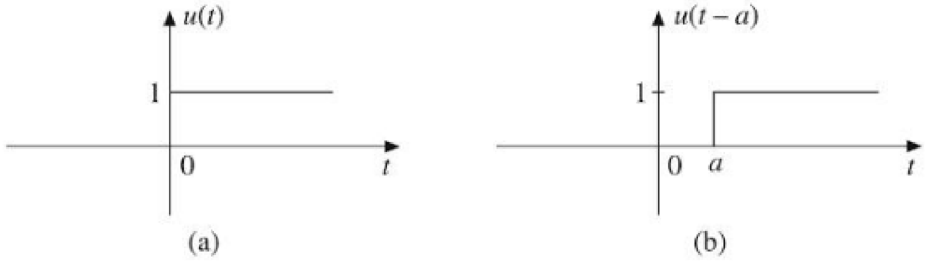

Figure 1 (a) Unit step function, (b) Delayed unit step function.

The discrete-time unit step sequence $u(n)$ is defined as:

$$u(n)=\begin{cases}1 & \text{for } n\geq 0\\ 0 & \text{for } n<0\end{cases}$$

The shifted version of the discrete-time unit step sequence $u(n-k)$ is defined as

$$u(n-k)=\begin{cases}1 & \text{for } n\geq k\\ 0 & \text{for } n<k\end{cases}$$

The graphical representations of $u(n)$ and $u(n-k)$ are shown in **Figure 2**[(a) and (b)].

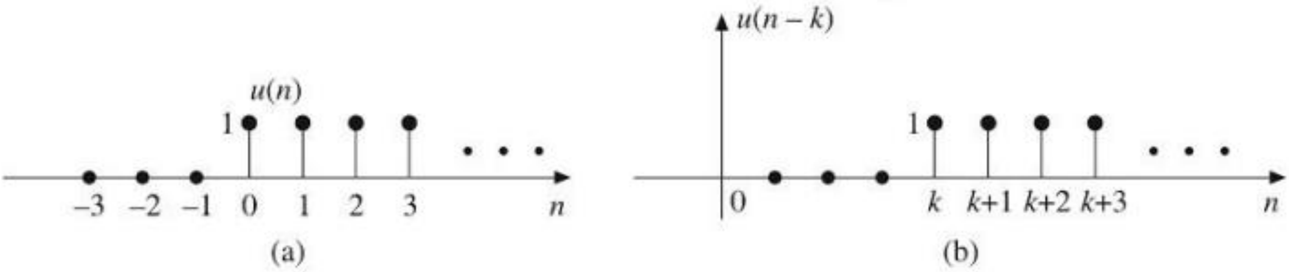

Figure 2 Discrete-time (a) Unit step function, (b) Shifted unit step function.

2.2 Unit Ramp Function

The continuous-time unit ramp sequence r(t) is that sequence which starts at t = 0 and increases linearly with time and is defined as:

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or $$r(t) = t\,u(t)$$

The unit ramp function has unit slope. It is a signal whose amplitude varies linearly. It can be obtained by integrating the unit step function. That means, a unit step signal can be obtained by differentiating the unit ramp signal.

i.e. $$r(t) = \int u(t)\,dt = \int dt = t \quad \text{for } t \geq 0$$

$$u(t) = \frac{d}{dt} r(t)$$

The delayed unit ramp signal $r(t-a)$ is given by

$$r(t-a) = \begin{cases} t-a & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

or $$r(t-a) = (t-a)\,u(t-a)$$

The graphical representations of $r(t)$ and $r(t-a)$ are shown in Figure 3 [(a) and (b)].

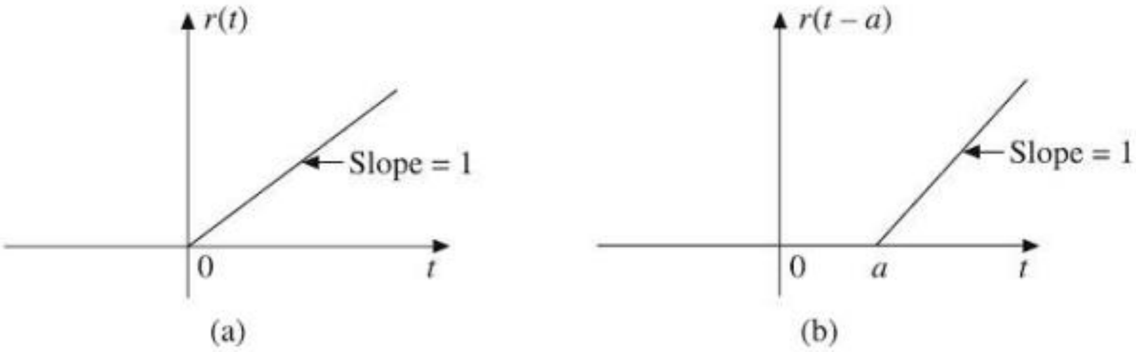

Figure 3 (a) Unit ramp signal, (b) Delayed unit ramp signal.

The discrete-time unit ramp sequence $r(n)$ is defined as

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or $$r(n) = n\,u(n)$$

The shifted version of the discrete-time unit-ramp sequence $r(n-k)$ is defined as

$$r(n-k) = \begin{cases} n-k & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

or $$r(n-k) = (n-k)\,u(n-k)$$

The graphical representations of $r(n)$ and $r(n-2)$ are shown in Figure 4 [(a) and (b)].

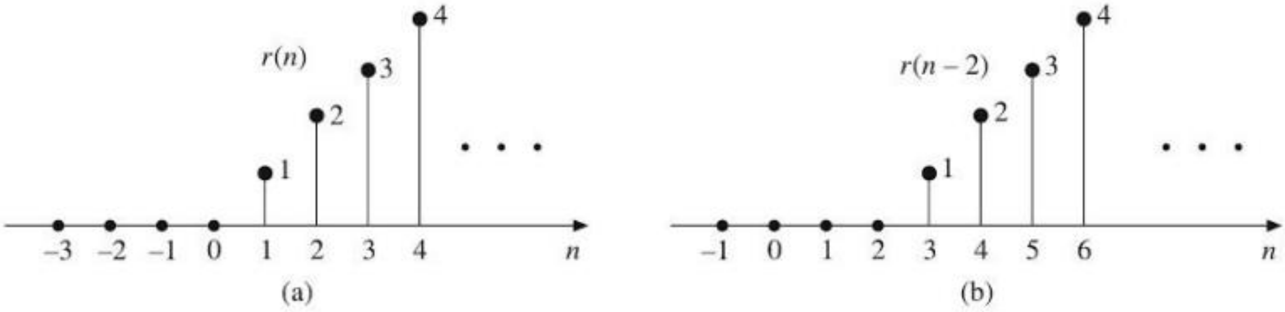

Figure 4 Discrete-time (a) Unit-ramp sequence, (b) Shifted-ramp sequence.

2.3 Unit Parabolic Function

The continuous-time unit parabolic function $p(t)$, also called unit acceleration signal starts at $t = 0$, and is defined as:

$$p(t) = \begin{cases} \dfrac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or
$$p(t) = \frac{t^2}{2} u(t)$$

The shifted version of the unit parabolic sequence $p(t-a)$ is given by

$$p(t-a) = \begin{cases} \dfrac{(t-a)^2}{2} & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

or
$$p(t-a) = \frac{(t-a)^2}{2} u(t-a)$$

The graphical representations of $p(t)$ and $p(t-a)$ are shown in Figure 5 [(a) and (b)].

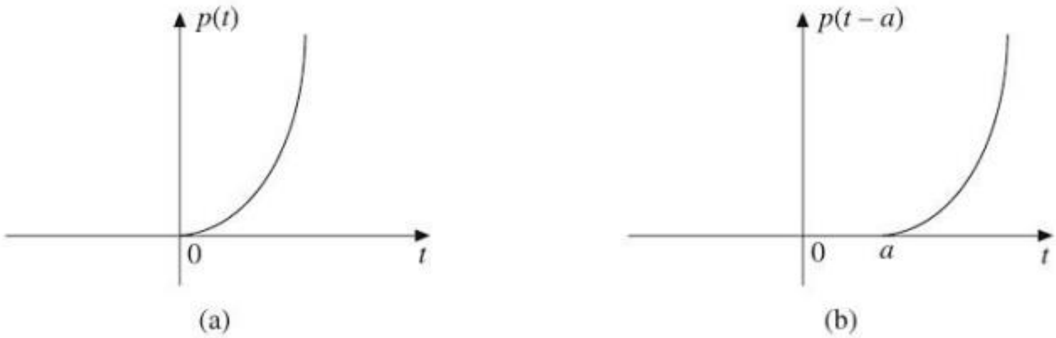

Figure 5 (a) Unit parabolic signal, (b) Delayed parabolic signal.

The unit parabolic function can be obtained by integrating the unit ramp function or double integrating the unit step function.

$$p(t) = \iint u(t)\, dt = \int r(t)\, dt = \int t\, dt = \frac{t^2}{2} \quad \text{for } t \geq 0$$

The ramp function is derivative of parabolic function and step function is double derivative of parabolic function

$$r(t) = \frac{d}{dt} p(t); \quad u(t) = \frac{d^2}{dt^2} p(t)$$

The discrete-time unit parabolic sequence $p(n)$ is defined as:

$$p(n) = \begin{cases} \frac{n^2}{2} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or

$$p(n) = \frac{n^2}{2} u(n)$$

The shifted version of the discrete-time unit parabolic sequence $p(n - k)$ is defined as:

$$p(n-k) = \begin{cases} \frac{(n-k)^2}{2} & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

or

$$p(n-k) = \frac{(n-k)^2}{2} u(n-k)$$

The graphical representations of $p(n)$ and $p(n - 3)$ are shown in Figure 6 [(a) and (b)].

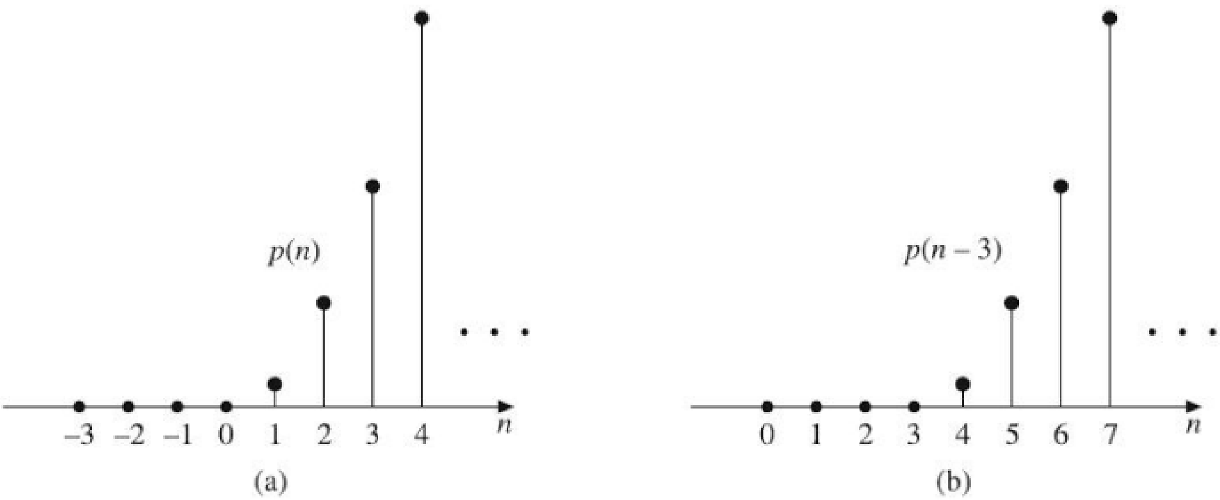

Figure 6 Discrete-time (a) Parabolic sequence, (b) Shifted parabolic sequence.

2.4 Unit Impulse Function

The unit impulse function is the most widely used elementary function used in the analysis of signals and systems. The continuous-time unit impulse function $\delta(t)$, also called Dirac delta function, plays an important role in signal analysis. It is defined as:

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases},$$

and the above Equation is also constrained to satisfy the identity as

$$\int_{-\infty}^{+\infty} \delta(t)\mathrm{d}t = 1$$

That is, the impulse function has zero amplitude everywhere except at $t = 0$. At $t = 0$, the amplitude is infinity so that the area under the curve is unity. $\delta(t)$ can be represented as a limiting case of a rectangular pulse function.

As shown in Figure 7 (a),

$$x(t) = \frac{1}{\Delta}[u(t) - u(t-\Delta)]$$

$$\delta(t) = \underset{\Delta \to 0}{\mathrm{Lt}}\; x(t) = \underset{\Delta \to 0}{\mathrm{Lt}}\; \frac{1}{\Delta}[u(t) - u(t-\Delta)]$$

A delayed unit impulse function $\delta(t-a)$ is defined as:

$$\delta(t-a) = \begin{cases} 1 & \text{for } t = a \\ 0 & \text{for } t \neq a \end{cases}$$

The graphical representations of $\delta(t)$ and $\delta(t-a)$ are shown in Figure 7 [(b) and (c)].

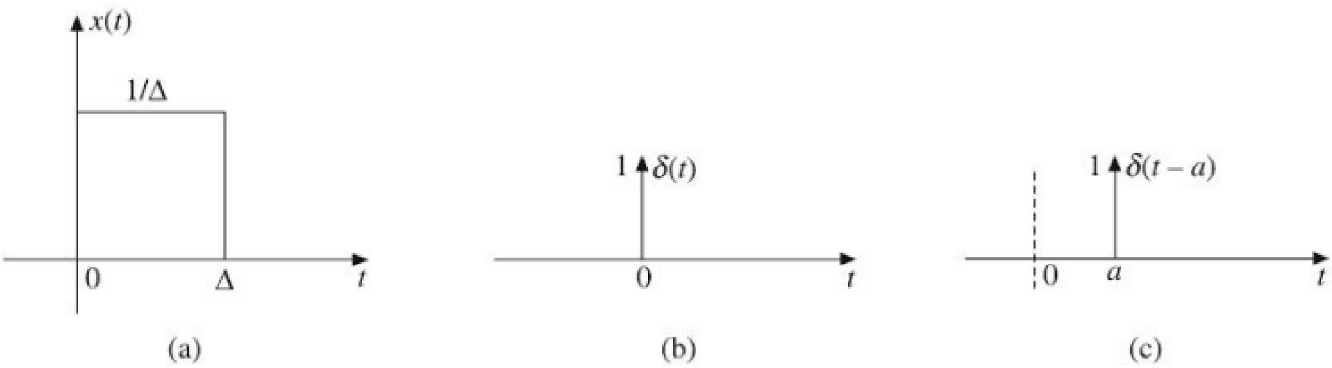

Figure 7 (a) $\delta(t)$ as limiting case of a pulse, (b) Unit impulse, (c) Delayed unit impulse.

If unit impulse function is assumed in the form of a pulse, then the following points may be observed about a unit impulse function.

(i) The width of the pulse is zero. This means the pulse exists only at $t = 0$.
(ii) The height of the pulse goes to infinity.
(iii) The area under the pulse curve is always unity.
(iv) The height of arrow indicates the total area under the impulse.

The integral of unit impulse function is a unit step function and the derivate of unit step function is a unit impulse function.

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt$$

and

$$\delta(t) = \frac{d}{dt} u(t)$$

Properties of continuous-time unit impulse function

1. It is an even function of time t, i.e. $\delta(t) = \delta(-t)$
2. $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0);\quad \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$
3. $\delta(at) = \frac{1}{|a|}\delta(t)$
4. $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0);\quad x(t)\,\delta(t) = x(0)\,\delta(t) = x(0)$
5. $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

The discrete-time unit impulse function $\delta(n)$, also called unit sample sequence, is defined as:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

The shifted unit impulse function $\delta(n-k)$ is defined as:

$$\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$$

The graphical representations of $\delta(n)$ and $\delta(n-3)$ are shown in Figure 8 [(a) and (b)].

Figure 8 Discrete-time (a) Unit sample sequence, (b) Delayed unit sample sequence.

Properties of discrete-time unit sample sequence

1. $\delta(n) = u(n) - u(n-1)$
2. $\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$
3. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$
4. $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

Relation between the unit sample sequence and the unit step sequence

The unit sample sequence $\delta(n)$ and the unit step sequence $u(n)$ are related as:

$$u(n) = \sum_{m=0}^{n} \delta(m), \quad \delta(n) = u(n) - u(n-1)$$

2.5 Sinusoidal Signal

A continuous-time sinusoidal signal in its most general form is given by

$$x(t) = A\sin(\omega t + \phi)$$

where

A = Amplitude
ω = Angular frequency in radians
ϕ = Phase angle in radians

Figure 9 shows the waveform of a sinusoidal signal. A sinusoidal signal is an example of a periodic signal. The time period of a continuous-time sinusoidal signal is given by

$$T = \frac{2\pi}{\omega}$$

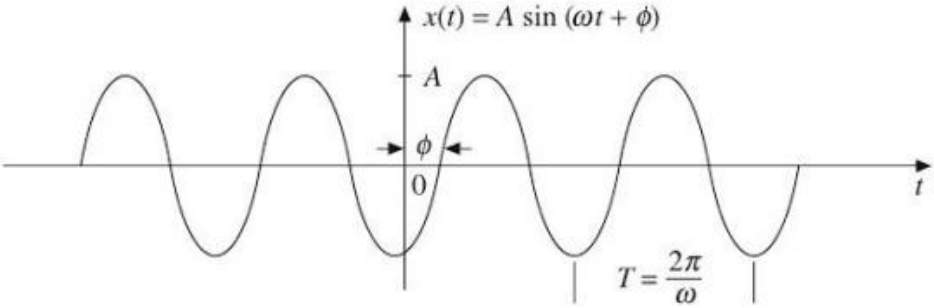

Figure 9 Sinusoidal waveform.

The discrete-time sinusoidal sequence is given by

$$x(n) = A\sin(\omega n + \phi)$$

where A is the amplitude, ω is angular frequency, ϕ is phase angle in radians and n is an integer.

The period of the discrete-time sinusoidal sequence is:

$$N = \frac{2\pi}{\omega} m$$

where N and m are integers.

All continuous-time sinusoidal signals are periodic but discrete-time sinusoidal sequences may or may not be periodic depending on the value of ω.

For a discrete-time signal to be periodic, the angular frequency ω must be a rational multiple of 2π.

The graphical representation of a discrete-time sinusoidal signal is shown in Figure10

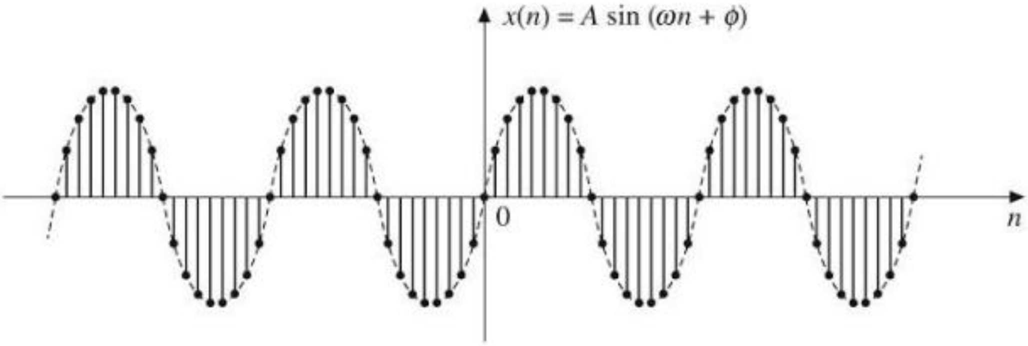

Figure10 Discrete-time sinusoidal signal.

2.6 Real Exponential Signal

A continuous-time real exponential signal has the general form as:

$$x(t) = Ae^{\alpha t}$$

where both A and α are real.

The parameter A is the amplitude of the exponential measured at $t = 0$. The parameter α can be either positive or negative. Depending on the value of α, we get different exponentials.

1. If $\alpha = 0$, the signal $x(t)$ is of constant amplitude for all times.
2. If α is positive, i.e. $\alpha > 0$, the signal $x(t)$ is a growing exponential signal.
3. If α is negative, i.e. $\alpha < 0$, the signal $x(t)$ is a decaying exponential signal.

These three waveforms are shown in Figure 11 [(a), (b) and (c)].

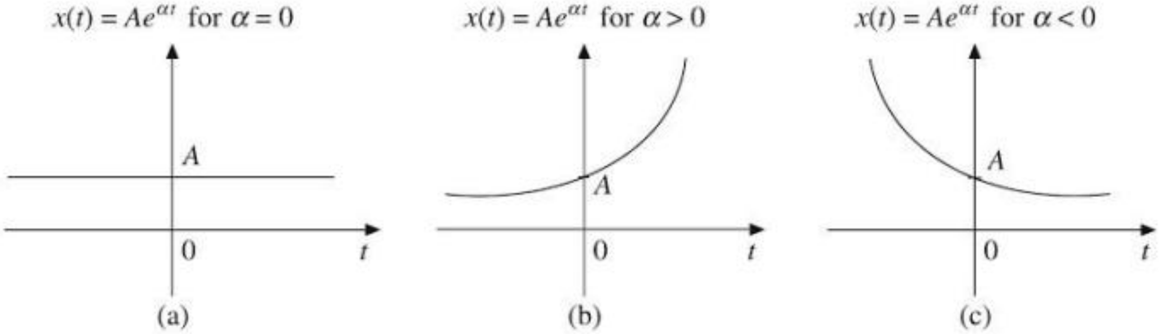

Figure 11 Continuous-time real exponential signals $x(t) = Ae^{\alpha t}$ for (a) $\alpha = 0$, (b) $\alpha > 0$, (c) $\alpha < 0$

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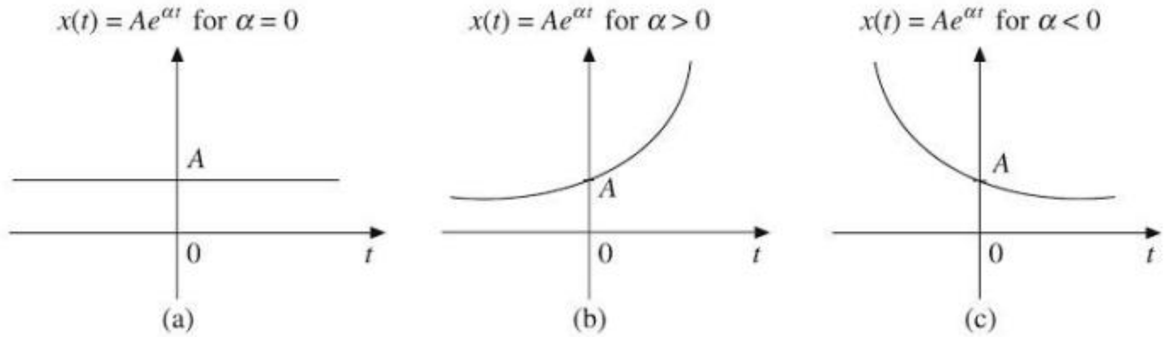

Figure 11 Continuous-time real exponential signals $x(t) = Ae^{\alpha t}$ for (a) $\alpha = 0$, (b) $\alpha > 0$, (c) $\alpha < 0$

The discrete-time real exponential sequence a^n is defined as:

$$x(n) = a^n \quad \text{for all } n$$

Figure12 illustrates different types of discrete-time exponential signals.

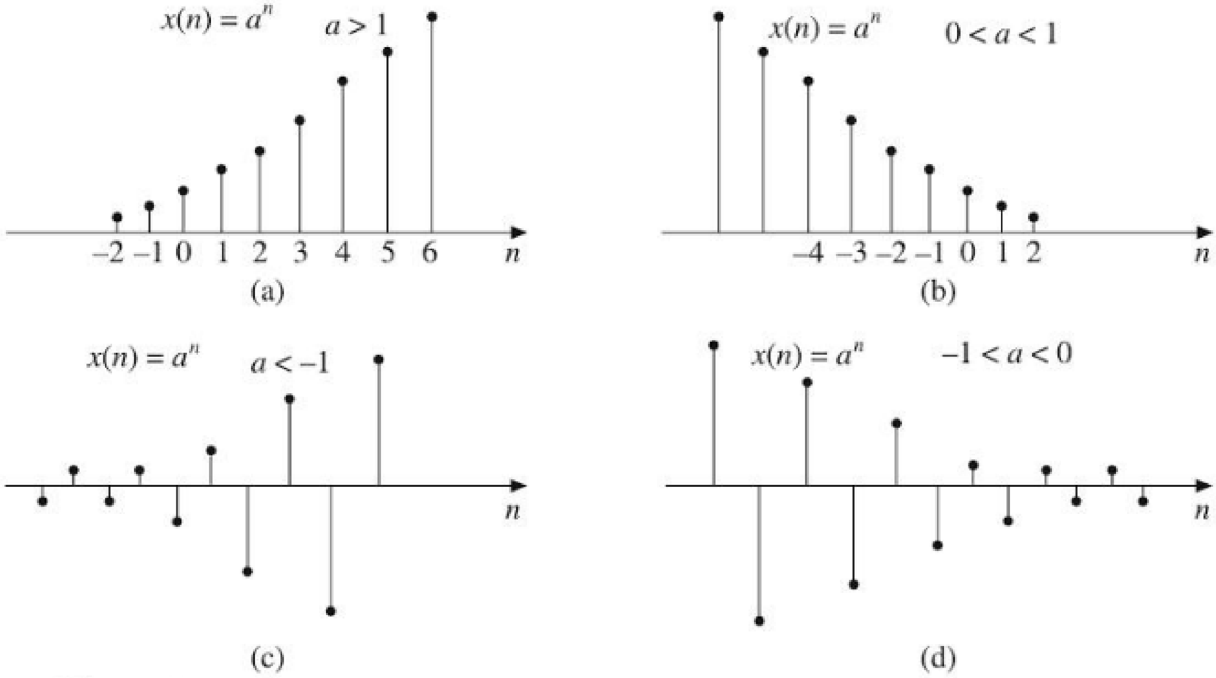

Figure12 Discrete-time exponential signal a^n for (a) $a > 1$, (b) $0 < a < 1$, (c) $a < -1$, (d) $-1 < a < 0$.

2.7 Complex Exponential Signal

The complex exponential signal has a general form as

$$x(t) = Ae^{st}$$

where A is the amplitude and s is a complex variable defined as

$$s = \sigma + j\omega$$

Therefore,

$$x(t) = Ae^{st} = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t}e^{j\omega t}$$

$$= Ae^{\sigma t}[\cos \omega t + j \sin \omega t]$$

Depending on the values of σ and ω, we get different waveforms as shown in Figure13

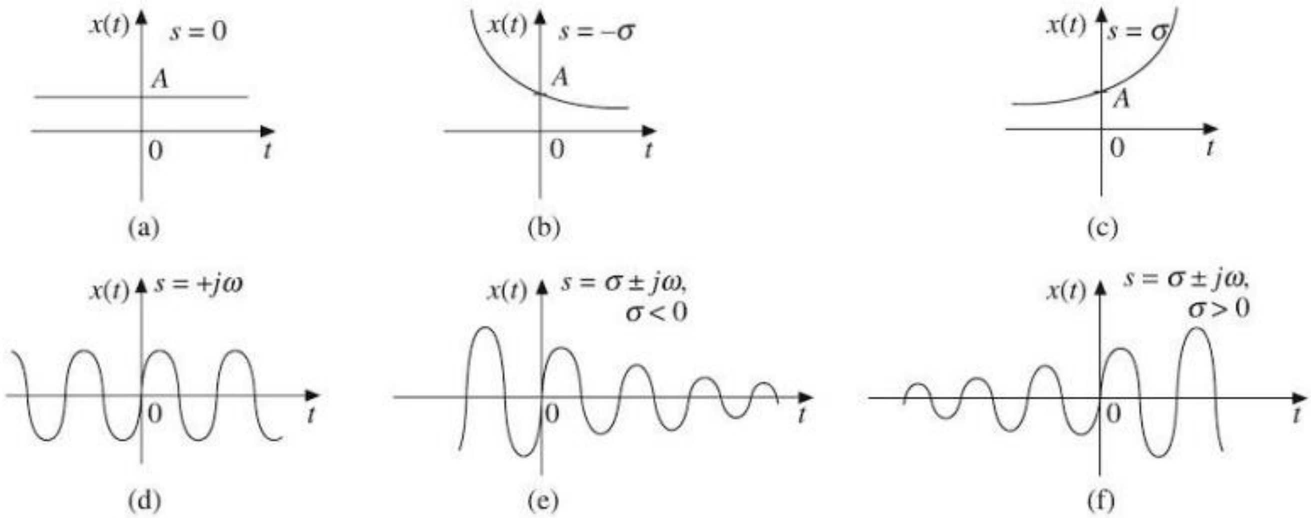

Figure13 Complex exponential signals.

The discrete-time complex exponential sequence is defined as

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$= a^n \cos(\omega_0 n + \phi) + ja^n \sin(\omega_0 n + \phi)$$

For $|a| = 1$, the real and imaginary parts of complex exponential sequence are sinusoidal.

For $|a| > 1$, the amplitude of the sinusoidal sequence exponentially grows

For $|a| < 1$, the amplitude of the sinusoidal sequence exponentially decays

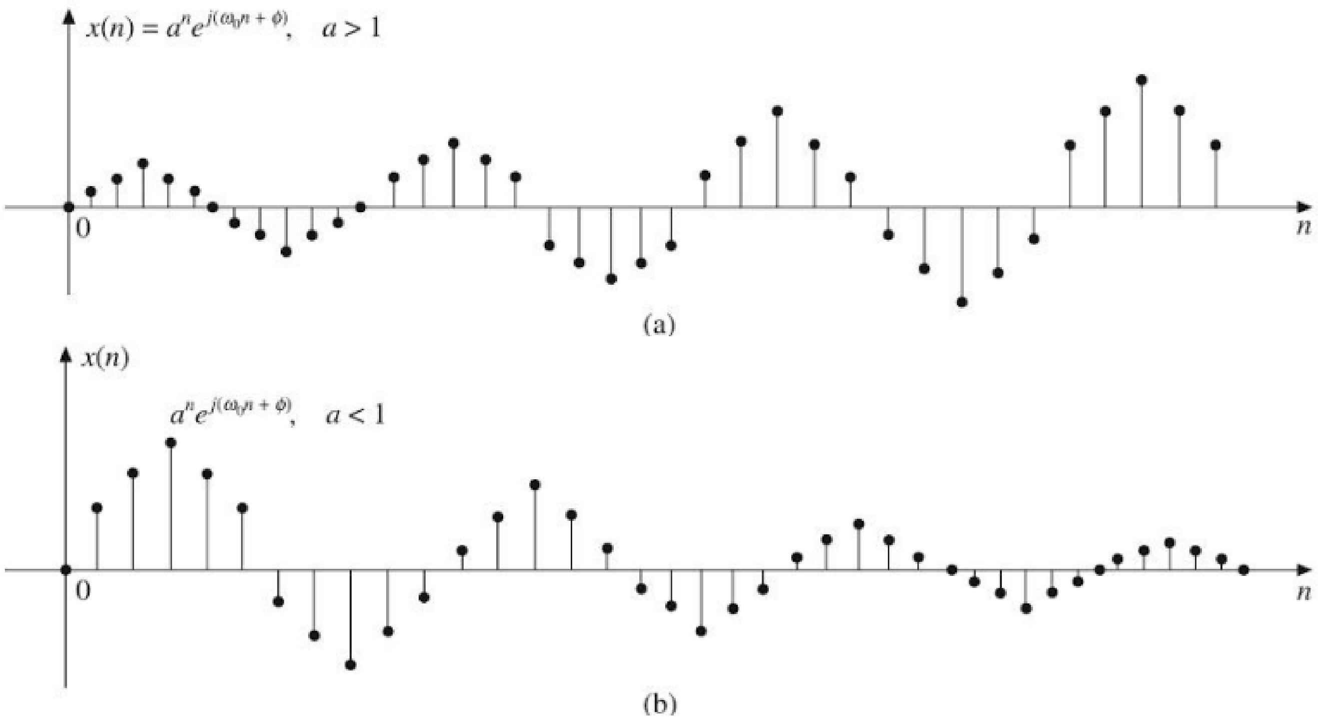

Figure 14 Complex exponential sequence $x(n) = a^n e^{j(\omega_0 n + \phi)}$ for (a) $a > 1$, (b) $a < 1$.