.Lecture five

Topics that must be covered in this lecture:

- ***Finite state automata with empty move.***
- ***Equivalence of NFA with and without ϵ move***

Finite state automata with empty move ϵ:

If the definition of a NFA is altered , so that moves from one state to another can be accomplished without any input we say that the automaton has ϵ -moves . NFA accepts W if there is some path labeled W from initial state to a final state. Of course, edges labeled ϵ may be included in the path, although the ϵ's do not appear explicitly in W.

More formally , a NFA M=(Q , ∑ , δ ,S ,F) has ϵ -moves if δ, instead of being a function Q x $\sum \rightarrow 2^Q$, is defined as a function Q x (∑ U{ ϵ }) $\rightarrow 2^Q$.

δ ' (Q, ϵ)=R(Q) or $\epsilon - closure(Q)$

δ ' (Q, ∑)=R(δ (R(Q), ∑))

Equivalence of NFA with and without ϵ move:

If there is NFA with ϵ , M = (Q, ∑, δ , q0, F) then there is NFA without empty move M^-= (Q^- , ∑, δ^-, q0 , F^-)

Define δ : Q×(∑ ∪ { ϵ }) → Q

By:

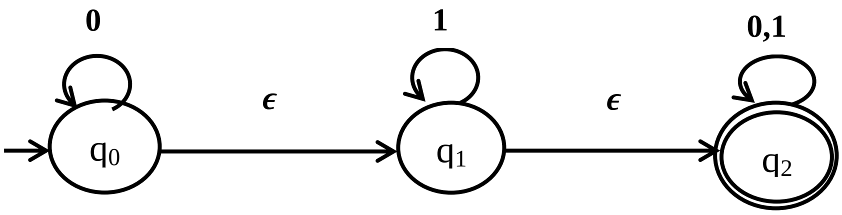
δ '(K,ε)=R(K) or ϵ -closure(K)

δ '(K,a)=R(δ (R(K),a))

The set of final states F^- is F ∪ {q} if R(q) ∈ F

Example1:

Convert the NFA with empty move into NFA without empty move.



Solution:

Note: 1- ϵ- closure ($\boldsymbol{\epsilon} *$): All the states that can be reached from a particular state only by seeing the ϵ symbol.

2-Every state on $\boldsymbol{\epsilon}$ goes to itself.

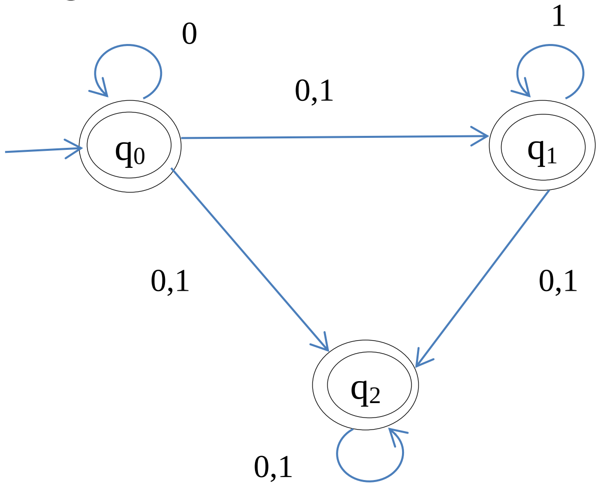
	$\boldsymbol{\epsilon} *$	0	$\boldsymbol{\epsilon} *$
q0	q0 q1 q2	q0 Φ q2	q0,q1, q2 - q2
q1	q1 q2	Φ q2	- q2
q2	q2	q2	q2

	$\boldsymbol{\epsilon} *$	1	$\boldsymbol{\epsilon} *$
q0	q0 q1 q2	Φ q1 q2	- q1,q2 q2
q1	q1 q2	q1 q2	q1,q2 q2
q2	q2	q2	q2

Transition table of NFA without $\boldsymbol{\epsilon}$:

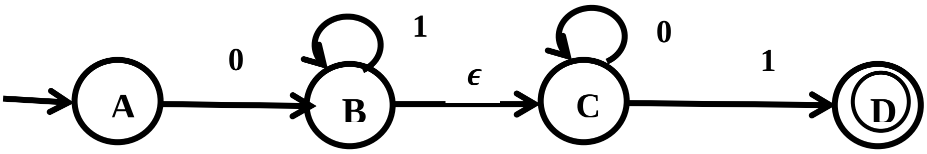
	0	1
q0	{q0,q1,q2}	{q1,q2}
q1	{q2}	{q1,q2}
q2	{q2}	{q2}

The state diagram of NFA without $\boldsymbol{\epsilon}$:



Example2:

Convert the NFA with empty move into NFA without empty move:



<u>Solution:</u>

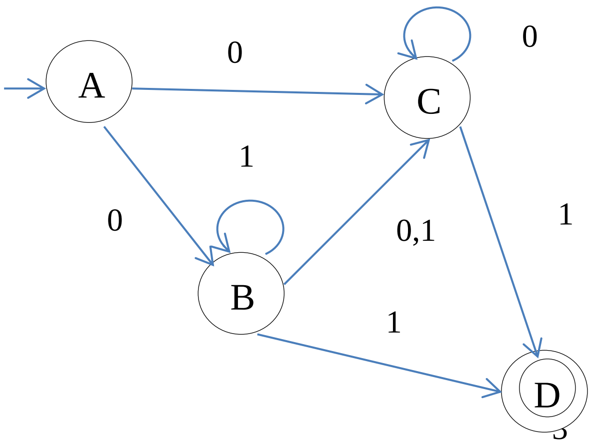
	$\epsilon *$	0	$\epsilon *$
A	A	B	B,C
B	B C	Φ **C**	– C
C	C	C	C
D	D	Φ	–

	$\epsilon *$	1	$\epsilon *$
A	A	Φ	–
B	B C	B D	B,C D
C	C	D	D
D	D	Φ	–

Transition table of NFA without ϵ:

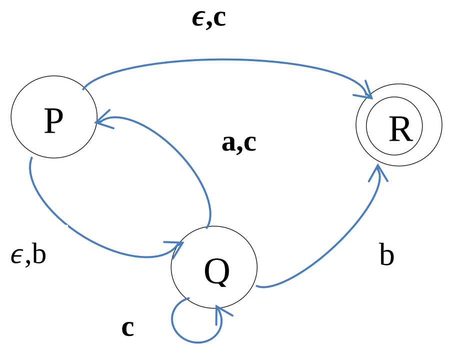
	0	1
A	{B,C}	Φ
B	{C}	{B,C,D}
C	{C}	{D}
D	Φ	Φ

The state diagram of NFA without ϵ:



Example3:

Convert the NFA with empty move into NFA without empty move:



<u>Solution:</u>

	$\epsilon*$	a	$\epsilon*$
P	P Q R	Φ P Q	- P,Q,R -
Q	Q	**P**	P,Q,R
R	R	Φ	-

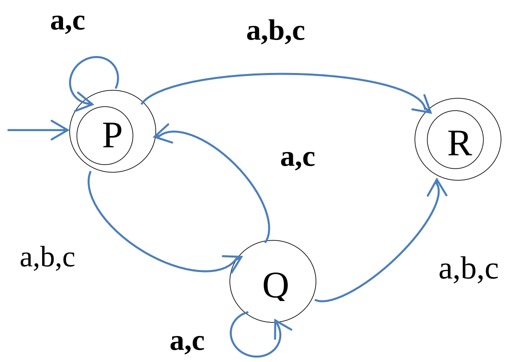
	$\epsilon*$	b	$\epsilon*$
P	P Q R	Q R Φ	Q R -
Q	Q	**R**	R
R	R	Q	_

	$\epsilon*$	c	$\epsilon*$
P	P Q R	R Q,P Φ	R P,Q,R -
Q	Q	**Q**	P,Q,R
R	R	Φ	_

Transition table of NFA without ϵ:

	a	b	c
P	{P,R,Q}	{Q,R}	{P,Q,R}
Q	{P,R,Q}	{R}	{P,Q,R}
R	Φ	Φ	Φ

The state diagram of NFA without ϵ:

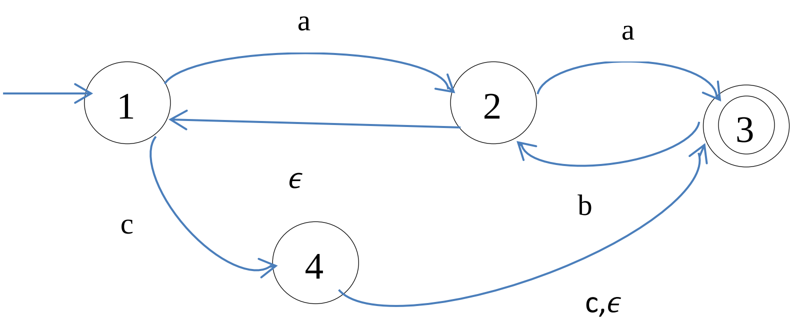


Conversion NFA with ϵ *to DFA*:

To construct DFA equivalent to the NFA with empty move apply the following steps:

1- Draw the NFA transition table and add new column contains ϵ-closure for all states in the NFA.
2- Start computing the DFA transition table from the first state and take the resulting states as the next state in each step.

Example1: construct DFA equivalent to the following NFA with empty move:



Solution:

	$\epsilon *$	a	$\epsilon *$
1	1	2	1,2
2	1 2	2 Φ	1,2 -
3	3	2	1,2
4	3 4	2 Φ	1,2 -

	$\epsilon *$	b	$\epsilon *$
1	1	Φ	-
2	1 2	Φ **3**	- 3
3	3	Φ	-
4	3 4	Φ Φ	- -

	$\epsilon *$	c	$\epsilon *$
1	1	4	3,4
2	1 2	**4** Φ	3,4
3	3	Φ	-
4	3 4	Φ 3	- 3

1- The transition table of NFA:

	a	b	c
1	1,2	Φ	3,4
2	1,2	3	3,4
3	1,2	Φ	Φ
4	1,2	Φ	3

2- The transition table of DFA:

	a	b	c
-1	1,2	D	3,4
1,2	1,2	3	3,4
+3,4	1,2	D	3
+3	1,2	D	D
D	D	D	D

3- The state diagram for DFA:

:طريقة اخرى

Solution:

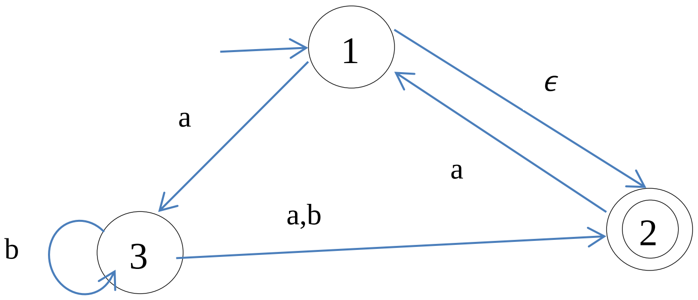
1- **The transition table of NFA with empty move :**

	a	b	c	ϵ-closure
1	2	-	4	1
2	-	3	-	2,1
3	2	-	-	3
4	-	-	3	4,3

2- **The transition table of DFA:**

	a ϵ *	b ϵ*	c ϵ*
-1	2,1	D	4,3
2,1	2,1	3	4,3
+4,3	2,1	D	3
+3	2,1	D	D
D	D	D	D

Example2: construct DFA equivalent to the following NFA with empty move:



Solution:

1- The transition table of NFA:

	a	b	ϵ-closure
1	3	-	1,2
2	1	-	2
3	2	2,3	3

2- The transition table of DFA:

	a ϵ *	b ϵ*
-,+ 1	3	D
3	2	2,3
+2	1,2	D
+2,3	1,2	2,3
+1,2	1,2,3	D
+1,2,3	1,2,3	2,3
D	D	D

3-The state diagram for DFA: