بسم الله الرحمن الرحيم

جامعة تكريت
كلية علوم الحاسوب والرياضيات

قسم: رياضيات

الوقت: 3 ساعات

اسئلة الامتحان التنافسي للطلبة المتقدمين للدراسات العليا (الماجستير)
للعام الدراسي 2015 – 2016

ملاحظات:

1. الامتحان مكون من أربع مجموعات.
   - مجموعة التفاضل والتكامل، المعادلات التفاضلية الاعتيادية والجزئية (20 درجة).
   - مجموعة أسس الرياضيات، التحليل، التبولوجي (25 درجة).
   - مجموعة الجبر الخطي، جبر الزمر والحلقات (20 درجة).
   - مجموعة الاحتمالات، الإحصاء الرياضي، التحليل العددي (25 درجة).

2. الوقت المحدد للمجموعة الأولى (36 دقيقة)، المجموعة الثانية (36 دقيقة)، المجموعة الثالثة (36 دقيقة)، المجموعة الرابعة (45 دقيقة).

3. يفرد دفتر خاص لكل مجموعة ويكتب عليه عنوان المجموعة وكذلك يكون لكل مشارك بالامتحان التنافسي اربع دفاتر امتحانية.

4. يحق للمشارك بامتحان التنافسي أن يستخدم الوقت الفاصل في أي مجموعة لحساب المجموعات الأخرى.

أ.م.د. حسن حسين إبراهيم
رئيس لجنة الامتحان التنافسي
Calculus

Q1 Mark two of the following by T if it is true or F when it is false
1- If f' is the derivative of f, then the derivative of the inverse of f is the inverse of f'.
2- $\cos(mx) \cos(nx) = \frac{1}{2}[(\cos(m+n)x - \cos(m-n)x)]$
3- To find the linear approximation to a function at $x = a$ you need to know the first derivative of that function.

Q2 Fill in the following blanks with correct answer
1- $\lim \left[ e^x - 1 \right] / x$ as $x$ approaches 0 is equal to ----------
   (a) 1  (b) 0  (c) is of the form 0/0 and cannot be calculated.
2- A critical number c of a function f is a number in the domain of f such that
   (a) $f'(c) = 0$  (b) $f'(c)$ is undefined  (c) (a) or (b)  (d) no one of them

Q3 Answer one of the following
1- Let $g(x) = \frac{x}{x-1}$ and $f(x) = \frac{1}{x+2}$ are functions define on $\mathbb{R}\{1\}, \mathbb{R}\{-2\}$ respectively. Find the domain of $gof$
2- Find the volume of the solid in the first quarter bounded by $x = 4 - y^2$
   and the plane $z = y$ and $x = 0, y = 0$

Ordinary and partial differential equations

Q1 Mark five of the following by T if it is true or F when it is false
1- Every linear combination for solutions of linear homogeneous differential equation is not a solution of this equation.
2- $\begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$ is not a solution of the equation $y' = Ay$ where $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
3- The system $y_1(t) = e^t, y_2(t) = 1 + \frac{e^{2t}}{2}$ is solution for initial value problem:
   $y_1' = y_1, y_2' = y_1^2$
   $y_1(0) = 1, y_2(0) = 3/2$
4- \( e^{\int 2x \, dx} \) is integral factor for the differential equation \( \frac{dy}{dx} + 2xy = xy^3 \)

5- The general solution for the equation \( y'' + 4y' + 3y = 8xe^x \) is \( y_c = Ae^{-x} + Be^{3x} \)

6- Let the initial value problem \( \frac{dy}{dt} = f(t,y) ; \quad y(t_0) = y_0 \)
   
   If \( f \) and \( \frac{\partial f}{\partial y} \) are continuous, then the initial value problem has a unique solution.

(5 marks)

Q2 Define two of the following
1- Partial differential equation
2- Linear differential equation
3- Initial Value Problem.

(4 marks)

Q3 Solve the following
1- Find the solution of the equation \( x^2 y'' + 2xy' - 1 = 0 \)
2- Determine whether the functions \( \sin(t) \) and \( \cos(t-\pi/2) \) are linearly independent or not on any arbitrary interval

(6 marks)
Foundation mathematics

Q1 Mark two of the following by T if it is true or F when it is false
1- If \( X, Y, Z, W \) are sets and \( f: X \to Y, g: Y \to Z, h: Z \to W \) are functions then 
\((h \circ g) \circ f = h \circ (g \circ f)\)
2- If \( \{A_i\}_i \) is family of subsets of \( X \) then 
\( f(\bigcap_i A_i) = \bigcap_i f(A_i) \)
3- let \( n > 0 \) be any natural number. On \( Z \), define the relation \( R \) by \( xRy \) if and only if \( n \) 
divide \( x-y \), then \( R \) is an equivalence relation

Q2 full one of the following blanks with correct answer
1- If \( f \circ g \) is one – to – one then \( g \) is -------------
   (a) one – to – one (b) onto (c) bounded (d) no one of them

2- let \( R \) be a relation on the natural numbers \( N \) define by \( xRy \) if and only if \( x \) divide \( y \) in 
\( N \) then \( R \) is -------------
   (a) symmetric (b) reflexive (c) antisymmetric (d) no one of them

Q3 Answer one of the following
1- Show that \( \bigcap_{n=1}^{\infty} \left( -\frac{1}{n}.1 + \frac{1}{n} \right) = [0, 1] \)
2- Let \( g \) and \( f \) are functions such that \( g \circ f \) is onto prove \( g \) is onto.

Analysis

Q1 Mark six of the following by T if it is true or F when it is false
1- The set \( S = \left\{ \frac{1}{2}, \frac{1}{1}, ..., \right\} \) is negligible set.
2- If \( \mu^*(S) = 0 \) then \( S \) is countable set, where \( \mu^*(S) \) is the outer measure of 
the set \( S \).
3- the set of irrational number is closed set in \( R \)
4- Every Cauchy sequence in \( R \) is converge.
5- If \( f(z) \) and \( \overline{f(z)} \) are entire function on the domain \( D \) then \( f \) is constant function
6- \( \cos(\overline{z}) = \cos y \cosh x + i \sin y \sinh x \).
7- the function \( f(z) = (z^2 - 2)e^{-x}e^{-iy} \) is not entire
8. \( \tan^{-1}z = (i/2)\log((i+z)/(i-z)) \)  

Q2 Full six of the following blanks with correct answer
1- If \( f \) is Riemann integrable function then \( |f| \) is --------- function.
   (a) continuous (b) not continuous (c) Riemann integrable  
   (d) not Riemann integrable

2- If \( f: [a, b] \to R \) is continuous function then \( f \) is ---------
   (a) differentiable (b) not differentiable (c) bounded (d) unbounded

3- Every increasing function is --------- function.
   (a) differentiable (b) Riemann integrable (c) continuous  
   (d) non of the above.

4- The equation \( x^2 = 8 \) has -----------
   (a) One positive rational root (b) one positive real root (c) two rational roots  
   (d) no one of them.

5- If \( e^z = 1 + \sqrt{3}i \) then \( z = \) -----------
   (a) 2i (b) i (c) 2 (d) -i

6- \( \log(-i) = \) -----------
   (a) -\( \pi i \) (b) \( \pi i \) (c) -\( \pi i/2 \) (d) \( \pi i/2 \)

7- \( (-i)^i = \) -----------
   (a) \( \pi i/2 \) (b) \( e^\pi i/2 \) (c) -\( \pi i/2 \) (d) \( e^{-\pi i/2} \)

Q3 Solve the following
1- Suppose that \( (X, d) \) is metric space where \( X \) is the set of rational numbers
   and \( d(x, y) = |x - y| \) \( x, y \in X \). Prove that the set \( E = \{ x \in X : 2 < x^2 < 3 \} \) is closed
   subset of \( X \).

2- prove that \( \frac{1}{2\pi i} \oint_c \frac{e^{az}}{z^2+1} \, dz = \sin \alpha \) where \( c: |z| = 3 \)

Q1 Mark three of the following by T if it is true or F when it is false
1- Every metric space is T2 – space  
2- Hilbert space is locally compact space  
3- Every compact set in T2 – space is closed  
4- Every locally connectedness is connected

Q2 full three of the following blanks with correct answer
1- Let \( E \) be a subset of a topological space \( (X, \tau) \). If \( E=d(E) \) then \( E \) is called ---------
(a) Scattered set (b) dense (c) perfect (d) clopen

2- Let $X=\{a, b, c\}$ and let $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ then $(X, \tau)$ is --------
   (a) Normally space (b) regular space (c) [CN] (d) [CR]

3- Any topological space is connectedness if and only if each of $X$ and $\emptyset$ is --------
   (a) Open set (b) closed set (c) perfect (d) clopen

4- Every metric space is -----------
   (a) [CN] (b) [CR] (c) [N] (d) [R]

Q3 Prove that if $E \subseteq (X, \tau)$ then $E^0 = \overline{E^c}$

(3marks)

(4marks)
Linear algebra
Q1 Mark two of the following by T if it is true or F when it is false
1- If V is n dimensional vector space and \( S = \{x_1, x_2, \ldots, x_n\} \) are linearly independent vectors in V then S is basis for V.
2- Let V be the space of all continuous functions on the interval \((-\infty, \infty)\) and \( W = \{f \in V : f(0) = 5\} \) then W is subspace of V.
3- If A, B are two matrices such that AB = 0 then either A = 0 or B = 0.

(2marks)

Q2 full one of the following blanks with correct answer
1- If A is a \( n \times n \) matrix has distinct eigenvalues then A is similar to \( \quad \) matrix.
   (a) non singular (b) singular (c) diagonal (d) no one of them.
2- If \( L: V \to R^5 \) is onto linear transformation and \( \text{dim}(\text{ker}(L)) = 2 \) then \( \text{dim}(V) = \quad \) .
   (a) 7 (b) 5 (c) 3 (d) 1.

(1marks)

Q3 Answer one of the following
1- If A is diagonalizable matrix prove \( A^k \) is diagonalizable matrix for any positive integer number k.
2- If \( \{X_1, X_2, X_3\} \) are linearly independent prove \( \{Y_1, Y_2, Y_3\} \) are linearly independent where \( Y_1 = X_1 + X_2 + X_3 \), \( Y_2 = X_2 + X_3 \), \( Y_3 = X_3 \).

(2marks)

Groups and rings
Q1 Mark five of the following by T if it is true or F when it is false
1- If \( G = (a) \) s.t O(G) = n, then for each positive integer k divides n, the group G has one subgroup of order k.
2- Let G be a finite group and let H be a subgroup of G, then O(G) divides O(H).
3- For any elements a, b in G and any integer n, then \( (a^{-1}ba)^n = a^{-n}b^n a^n \).
4- The ideal generated by 2 is maximal in \( Z_n \) for any odd positive integer \( n \geq 2 \).
5- A ring \( Z_5 \) is an integral domain.
6- Let \( f: R_1 \to R_2 \) is a homomorphism of rings, if \( I \) is an ideal of \( R_2 \) then \( f^{-1}(I) \) is an ideal \( R_1 \).

Q2 Full four of the following blanks with correct answer

1- Let \( a \) be an element in a group then the subgroup \( (a^{12}) \cap (a^{18}) = \) .................
   (a) \( a^{36} \) (b) \( a^{12} \) (c) \( a^{18} \) (d) \( a^6 \)

2- Let \( H \) and \( K \) be two subgroups of a group \( G \) s.t \( D = H \cap K \neq \{e\} \) if \( O(H)=14 \) and \( O(K)=35 \) then \( O(D)=\) .................
   (a) 49 (b) 14 (c) 35 (d) 7

3- Intersection of two subrings is ...........
   a- not subring b- subring c- ideal d – center of subring

4- Let \( f: R \to \hat{R} \) is a homomorphism of rings, if \( I \) is an ideal of \( R \) and \( f \) is ...........
   function, then \( f(I) \) is an ideal of \( \hat{R} \).
   a- one-one b- onto c- identity d- inverse

5- The all zero divisor elements of the ring \( \mathbb{Z}_{12} \) are ............
   a-\{2,3,4,6,8\} b-\{4,6,8,9,10\} c-\{2,3,8,9,10\} d-\{2,3,4,6,8,9,10\}

Q3 Solve the following

1- Let \( G \) be a group with exactly 4 elements . prove that \( G \) is abelian .

2- If \( U \) is an ideal of a ring \( R \); let \( r(U) = \{x \in R : xu = 0 \text{ for all } u \in U\} \). Prove that \( r(U) \) is an ideal of \( R \)
Probability
Q1 Mark two of the following by T if it is true or F when it is false
1- If \( A \) anb \( B \) are two Events then \( Pr(A \cap B) = Pr(A) + Pr(B) \)
2- If \( Pr(X) = 1 \) then \( Pr(X^c) = 0.5 \).
3- \( Pr(a \leq X \leq b) = Pr(X > a) + Pr(X > b) \)

(2 marks)

Q2 full one of the following blanks with correct answer
1- If \( X \sim N (\mu, \sigma^2) \) then \( Var(X) = \cdots \)
2- If \( X \sim \chi^2(\alpha) \) then \( M_X(t) = \cdots \)

(1 mark)

Q3 Answer one of the following
1- If \( A \) and \( B \) are two events prove that \( Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B) \).
2- If \( X_i ; i = 1, 2, ..., n \) are random sample with size \( n \) where \( X_i = i \ for \ each \ i = 1, 2, ..., n \) prove that \( E(X) = \frac{n+1}{2} \).

(2 marks)

Mathematical statistic
Q1 Mark three of the following by T if it is true or F when it is false
1- The Normal Distribution is continuous Distribution.
2- The distribution function of discrete r. v. \( X \) is equal's \( Pr(X \leq x) = \int_{-\infty}^{x} f(x)dx \).
3- If \( X \sim Poisson(p) \) then \( E(X) = p \)
4- If \( X \sim Bernoulli(p) \) then \( E(X) = 1 - p \)

(3 marks)

Q2 full three of the following blanks with correct answer
1- If \( X \sim N (\mu, \sigma^2) \) then \( Var(X) = \cdots \)
2- If \( X \sim \chi^2 \) then \( Y = \sqrt{X} \sim \cdots \)
3- If \( X \sim \text{Geometric}(p) \) then \( f(x) = \cdots \)
4- Let \( X_i \) are independent r.vs. and \( X_i \sim Bernoulli (p) \) i = 1, 2, ..., \( n \), then \( Y = \sum_{i=1}^{n} X_i \sim \cdots \)

(3 marks)
Q3 Solve the following

1- If \( X_i; \ i = 1, 2, \ldots, n \) are independent r. vs. such that \( X_i \sim \text{Exp} (\beta) \) for each \( i \) prove that \( Y \sim \Gamma(n, \beta) \) where \( Y = \sum_{i=1}^{n} X_i \).

(4 marks)

Numerical analysis

Q1 Mark three of the following by T if it is true or F when it is false

1- The convergence speed of False Position method is Linear.
2- In the interpolation by lagrange method then the different between points equals.
3- The Bessel method Used when value be in the first table.
4- The Bool method in numerical integral used when the number of point is seven.

(3 marks)

Q2 full three of the following blanks with correct answer

1- A formula of Jaccobi method to solve the system of equations is

\[
\begin{align*}
\text{a-} x_i &= \left[ b_i - \sum_{j=1}^{n} a_{ij} x_j \right] / a_{ii} \\
\text{b-} x_i &= \left[ b_i - \sum_{j=1}^{n} a_{ij} x_j \right] / a_{jj} \\
\text{c-} x_i &= \left[ b_i - \sum_{j=1}^{n} a_{ij} x_j \right] / a_{ii} \\
\text{d-} x_i &= \left[ b_i - \sum_{j=1}^{n} a_{ij} x_j \right] / a_{ii}
\end{align*}
\]

2- In the Romberg method \( R(k, i) = \frac{\ldots}{\ldots}, \ k=2,3,\ldots,n \)

\[
\begin{align*}
\text{a-} R(k, i) &= \frac{1}{2} \left[ R(k-1, i) + h_{k-1} \sum_{i=1}^{2k-2} f(a + (i - \frac{1}{2})h_{k-1}) \right] \\
\text{b-} R(k, i) &= \frac{4^{i-2} R(k-1, i - 1) - R(K-1, i - 1)}{4^{i-2} - 1} \\
\text{c-} R(k, i) &= \frac{1}{2} \left[ R(k-1, i) + h_{k-1} \sum_{i=1}^{2k-2} f(a + (i - \frac{1}{2})h_{k-1}) \right] \\
\text{d-} R(k, i) &= \frac{4^{i-1} R(k-1, i - 1) - R(K-1, i - 1)}{4^{i-1} - 1}
\end{align*}
\]

3- \( \delta^3 f_i = \)

\[
\begin{align*}
\text{a-} f_{\frac{i+3}{2}} - 3f_{\frac{i+1}{2}} + 3f_{\frac{i-1}{2}} - f_{\frac{i-3}{2}} \\
\text{b-} f_{\frac{i+3}{2}} + 3f_{\frac{i-1}{2}} - 3f_{\frac{i+1}{2}} - f_{\frac{i-3}{2}}
\end{align*}
\]
c- $f \left( 1 + \frac{3}{2} \right) - 3f \left( 1 + \frac{1}{2} \right) + 3f \left( 1 + \frac{1}{2} \right) + f \left( 1 - \frac{1}{2} \right)$

4- If you have the following data

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-0.5</td>
<td>0.5</td>
<td>1</td>
<td>1.3</td>
<td>1.5</td>
</tr>
</tbody>
</table>

and you want to find $f(1.4)$ by use Bessel formula, then the value of $x_0$ is

a- 0   
b- 1   
c- 2   
d- 0.5

(3 marks)

Q3 Solve one of the following

1- What is the convergence conditions for the Fixed point iterative method to solve system of nonlinear equations.

2- Derive the formula of Modified Euler method for solve ordinary Differential Equations.

(4 marks)