بسم الله الرحمن الرحيم

جامعة تكريت
كلية علوم الحاسوب والرياضيات

القسم: رياضيات
الوقت: 3 ساعات

اسئلة الامتحان التنافسي للطلبة المتقدمين للدراسات العليا (الماجستير)
للعام الدراسي 2015 - 2016

ملاحظات:

1. الاسئلة مكونة من اربع مجاميع وهي:
   - مجموعة التفاضل والتكامل ، المعادلات التفاضلية الاعتيادية والجزئية (20 درجة)
   - مجموعة اسس الرياضيات ، التحليل الرياضي ، التحليل العقدي ، التبولوجي (35 درجة)
   - مجموعة الجبر الخطي ، جبر الزمر والحلقات (20 درجة)
   - مجموعة الإحصاء البيانات والرياضيات الاحصائية (25 درجة)
2. الوقت المحدد للمجموعة الأولى (36 دقيقة) ، المجموعة الثانية (36 دقيقة)
   - المجموعة الثالثة (36 دقيقة) ، المجموعة الرابعة (36 دقيقة)
   - يفرد دفتر خاص لكل مجموعة ويكتب عليه عنوان المجموعة وبذلك يكون لكل مشارك
     بالامتحان التنافسي اربع دفاتر امتحانية.
4. يحق للمشارك بالامتحان التنافسي ان يستخدم الوقت الفاضل في أي مجموعة لحساب
   المجموع الأخرى.

أ.م.د.حسن حسين إبراهيم
رئيس لجنة الامتحان التنافسي
Calculus

Q1 Mark two of the following by T if it is true or F when it is false

1. If a function f is continuous at x = a, then it has a tangent line at x = a.
2. f(g(x)) = g(f(x)) for any two functions f and g
3. The derivative of f(x) = a^x with respect to x, where a is a constant, is x a^{-1}.

(2marks)

Q2 full one of the following blanks with correct answer

1. Let the closed interval [a, b] be the domain of a function f. The domain of f(x - 3)
   is given by ----------
   (a) (a, b)  (b) [a, b]  (c) [a - 3, b - 3]  (d) [a + 3, b + 3]

2. \( \lim_{x \to \infty} \left( \frac{\cos(x) - 1}{x} \right) = ----------
   (a) 1  (b) 0  (c) -1  (d) \infty

(1marks)

Q3 Answer one of the following

1. For f(x) = ln x, find the first derivative of the composite function defined by
   \( F(x) = f \circ f(x) \)

2. Reverse the order of the double integral \( \int_0^2 \int_1^e dy \, dx \).

(2marks)

Ordinary and partial differential equations

Q1 Mark five of the following by T if it is true or F when it is false

1. The partial differential equation contain only one independent variable and only
   one dependent variable
2. The differential equation \( x^2 y'' + xy' - 9y = 5x \) is Euler equation.
3. The general solution for the linear homogeneous differential equation of order n
   with constant coefficient when the roots are distinct be in the form
   \( y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \ldots + c_n e^{m_n x} \) Where \( c_1, c_2, \ldots, c_n \) are arbitrary constants.
4. The vector functions \( u_1, u_2, \ldots, u_n \) are linearly dependent if \( W(u_1, u_2, \ldots, u_n)(t) = 0 \).
5. The integral factor for the differential equation \( \frac{dy}{dx} + 2xy = xy^3 \) is \( e^{\int 2xdx} \)
6. The equation \( (x^3 + y^3)dx - 2xy^2 dy = 0 \) is homogeneous equation of degree three
Q2 Define two of the following  
1- Initial condition  2- Ordinary Linear Equation.  3- Rank of Equation.  (4marks)

Q3 Solve the following  
1- Explain how the initial value problem  
\[ y''' - 2x^2 \, y'' + 5xy' + 3y = \cos x \]
\[ y(-2) = 3 \]
\[ y'(-2) = 1 \]
\[ y''(-2) = -2 \]
has the unique solution in interval \( R = \{ x : -\infty < x < \infty \} \).

2- Solve the differential equation \( y'' + y = \sec(x) \)  (6marks)
Foundation mathematics

Q1 Mark two of the following by T if it is true or F when it is false
1- \((N, \leq)\) is well ordered
2- If \(p, n, m \in N\) then \((p=nm) \land (n \neq 1) \rightarrow (p < m)\)
3- Let \(A, B\) are countable sets then \(A \cup B\) is uncountable set.

Q2 full one of the following blanks with correct answer
1- If \(f: X \rightarrow Y\) is one – to – one mapping and \(B \subseteq Y\) then
   (a) \(f(f^{-1}(B)) = f(X) \cup B\)  (b) \(f(f^{-1}(B)) = f(X) \cap B\)  (c) \(f(f^{-1}(B)) = B\)
2- Let \(f: X \rightarrow Y\) be a function and let \(A \subseteq X, B \subseteq Y\) then
   (a) \(A \subseteq f^{-1}(f(A))\)  (b) \(f^{-1}(f(A)) \subseteq A\)  (c) \(A = f^{-1}(f(A))\)

Q3 Answer one of the following
1- Let \(n \in Z^+\) prove that \(\equiv_n\) is equivalence relation on \(Z\)
2- prove that \(\leq\) is a partial order relation on \(N\).

Analysis

Q1 Mark six of the following by T if it is true or F when it is false
1- If \(a_n = \left(1 + \frac{1}{n}\right)^n\), \(n=1,2,3,...\) then the sequence \(\{a_n\}\) is converge .
2- The set of rational numbers is closed.
3- If \(X\) is metric space then \(f: X \rightarrow X\) is continuous function if and only if
   \(\lim_{n \to \infty} \{f(x_n)\} = f\left(\lim_{n \to \infty} \{x_n\}\right)\)
4- If \(\{f_n\}\) is sequence of differentiable functions that converge to \(f\) then \(f\) is differentiable function.
5- \(\sqrt{2}|z| \leq |ReZ| + |ImZ|\)
6- \(ArgZ = -Arg\left(\frac{1}{z}\right)\)
7- The function \(f(z) = \bar{z}\) is differentiable every where .
8- The function \(f(z) = (z^2 - 2)e^{-x}e^{-iy}\) is not entire
Q2 Full six of the following blanks with correct answer
1- Every monotonic function is -------------- function.
   (a) differentiable (b) Riemann integrable (c) continuous (d) no one of them.
2- The set $S \subseteq \mathbb{R}^n$ is compact set if and only if $S$ is --------------
   (a) Closed (b) bounded (c) closed and bounded (d) open and bounded
3- If $\alpha$ is positive real number and $n$ is positive integer number then the equation $x^n = \alpha$ has
   -------------- positive real solution.
   (a) One  (b) no  (c) countable set of  (d) no one of them
4- If $\{f_n\}$ is -------------- to $f$ then $\lim \int f_n = \int \lim f_n$.
   (a) pointwise converge  (b)uniform converge (c) increasing sequence
   (d) decreasing sequence
5- The set of irrational numbers is -------------- set.
   (a) countable finite (b) countable infinite (c) uncountable (d) negligible
6- The function $f$ is Riemann integrable if and only if the set of discontinuous points for $f$ is
   -------------- set.
   (a) Finite (b) infinite (c) closed (d) negligible
7- For every complex number $z = x+iy$ we have $|\sinh y|$ -------------- $|\sin z|
   (a) \leq  (b) \geq  (c) =  (d) \neq$

Q3 Solve the following
1- Suppose that $E$ and $G$ are sets in a metric space $(X,d)$ , where $G$ is open set. Prove that if $G \cap \overline{E} \neq \emptyset$ then $G \cap E \neq \emptyset$.
2- prove that $x^2 - y^2 = 1$ can be written as $z^2 + \overline{z}^2 = 2$ , where $z=x+iy$.

Topology
Q1 Mark three of the following by T if it is true or F when it is false
1- In a $T_2$ - space the convergent sequence is convergent to a unique point
2- In $T_2$ – space every finite set has a limited point
3- If $\overline{f(E)} \subseteq \overline{f(E)}$ then $f : (X, \tau) \to (X', \tau')$ is closed function such that
   $(X', \tau')$ is a subspace of topological space $(X, \tau)$
4- Every Hilbert space is a topological space

Q2 full three of the following blanks with correct answer
1- Every sequentially compact is -----------
   (a) Compact (b) locally compact (c) count ably compact (d) no one of them
2- If $E \cap \overline{d(E)} = \emptyset$ we say that $E$ is -----------
   (a) Isolated (b) superset (c) perfect (d) ) no one of them
3- Any topological space is connectedness if and only if each if $X$ and $\emptyset$ is -----------
   (a) Open set (b) closed set (c) perfect (d) clopen
4- Every compact Hausdorff space is --------
   (a) Normal  (b) [CN]  (c) Regular  (d) no one of them  (3marks)

Q3 Prove that $A \cup A'$ is closed set where $A' = d(A)$  (4marks)
Linear algebra
Q1 Mark two of the following by T if it is true or F when it is false
1- If A,B,C are matrices such that AB=AC then B=C .
2- If L:V→ W is linear transformation and dim(V)=dim(W)=n then L is one – to – one if and only if L is onto.
3- If A is n × n matrix then Ax=0 has non zero solution if and only if |A| ≠ 0, where|A| is the determinate of A.
(2marks)

Q2 full one of the following blanks with correct answer
1- The vectors \( x_1 = (4, 2, 6, -8) \) and \( x_2 = (-2, 3, -1, -1) \) in \( \mathbb{R}^4 \) are -------.
   (a) orthogonal (b) opposite to each other (c) in the same direction
   (d) no one of them .
2- If \( L: R^6 \to R^6 \) is linear transformation and dim(rang(L))=5 then \( \text{dim(ker(L))} = \) -------.
   (a) 1 (b) 3 (c) 2 (d) 8 .
(1marks)

Q3 Answer one of the following
1- If A is \( n \times n \) matrix prove \( A=S+K \), where \( S^T = S \), \( K^T = -K \).
2- Find basis for \( W=\left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} : b = a + c, a, b, c \in \mathbb{R} \right\} \).
(2marks)

Groups and rings
Q1 Mark five of the following by T if it is true or F when it is false
1- Let \( G \) be a group and \( a \in G \) . if \( a^m = e \) then O(a) divides m.
2- Let p be a prime number and n , m be positive integers s.t p divides nm, then p divides n and m .
3- If \( G=(a) \) is a group such that O(G)=n , then for each positive integer k divides n , the group G has one subgroup of order k .
4- Let A be a finite ring and \( a, b \in A \) s.t \( ab=1 \) then \( ba=1 \)
5- The ideal generated by 2 is maximal in \( Z_n \) for any odd positive integer \( n \geq 2 \)
6- The Centre of ring is an ideal
(5 marks)
Q2 Full four of the following blanks with correct answer

1- Let H be a subgroup of a group G, then H is normal iff …………….. for each g in G

2- Let a be an element in a group G s.t \( O(a) = 20 \) then \( O(a^6) = \) …………….. 

3- Let a, b be two elements in a group G s.t \( O(a) = n \) and \( O(b) = m \) and \( \gcd(n,m) = 1 \), then \( H = (a) \cap (b) = \) …………….. 

4- The all zero devisor elements of the ring \( \mathbb{Z}_{12} \) are …………….. 

5- Let \( f : R_1 \rightarrow R_2 \) be a homomorphism of rings, then \( \ker f = \{0\} \) if and only if \( f \) is …………….. 

Q3 Solve the following

1- If G is a group such that \( (ab)^2 = (a^2b^2) \) \( \forall a, b \in G \). Prove that G is commutative. 

2- Let I, J be two distinct maximal ideal of a commutative ring A with 1, prove that \( IJ = I \cap J \). 

(4 marks) 

(6 marks)
Probability

Q1 Mark two of the following by T if it is true or F when it is false
1. If \( A \text{ and } B \) are two Events then \( P(A \cup B) = P(A) + P(B) \)
2. If \( X \) and \( Y \) are two independent random variables then \( P(X \setminus Y) = P(X) \)
3. If \( A \text{ and } B \) are two Events where \( P(A) = 0.3 \) and \( P(B) = 0.4 \)
then \( P(A \cup B) = 0.12 \)

Q2 Full one of the following blanks with correct answer
1. If \( X \sim \text{Geometric} (p) \) then \( E(X) = \) __________
2. The moment generated function of r.v. \( X \) is \( M_X(t) = \) __________

Q3 Answer one of the following
1. If \( P(A) = a \) and \( P(B) = b \) find \( P(\overline{A} \cap \overline{B}) \)
2. If \( X \sim \text{Binomial} (n, p) \) find the probability distributed of r.v. \( Y = n - X \).

Mathematical statistic

Q1 Mark three of the following by T if it is true or F when it is false
1. The Poisson Distribution is continuous.
2. The distribution function of r.v. \( X \) is equal's to \( \int_{-\infty}^{\infty} f(x) \, dx \)
3. The characteristic function of r.v. \( X \) is equal's to \( E(e^{itx}) \)
4. If \( X \sim \text{binomial} (n, p) \) if \( XY = \frac{x-np}{np(1-p)} \sim N(0, 1) \) where \( p \) is larger and \( n \) is smaller

Q2 Full three of the following blanks with correct answer
1. The r.v.s \( X_n \) Converges in Distribution to r.v \( X \) if __________
2. Two r.v.s are Equivalent if __________
3. Let \( A \subseteq \Omega \) and function \( I_A: \Omega \to \{0, 1\} \) which is define by_______ is called characteristic function.
4. The density function of r.v \( X \sim \text{Uniform} (a, b) \) is equal's __________

Q3 Prove that if \( X \sim N(0, 1) \) then \( X^2 \sim \chi^2(1) \).
Numerical analysis

Q1 Mark three of the following by T if it is true or F when it is false

1- The Inherent Error is the error product by substitute the infinite equation by finite equation.
2- The Secant method depended on the different of signal.
3- The convergence of Newton-Raphson method is global.
4- The Partial Pivot is substitute the row between them.

Q2 Full three of the following blanks with correct answer

1- A formula of Newton-Raphson method to find the square root is
\[ a- \ x_{i+1} = \frac{x_i^2 + A}{2x_i} \quad b- \ x_{i+1} = \frac{x_i^2 - A}{2x_i} \]

\[ c- \ x_{i+1} = \frac{1}{2}(x_i - \frac{A}{x_i}) \quad d- \ x_{i+1} = \frac{1}{2}(2x_i - \frac{A}{x_i}) \]

2- The enough condition to convergent the fixed point method is
\[ a- |g(x)| \leq L, L > 0 \quad b- |g(x)| \leq L, L > 1 \]
\[ c- |g(x)| \leq L, L < 1 \quad d- |g(x)| \leq L, L < 0 \]

3- If we have the following data

<table>
<thead>
<tr>
<th>x</th>
<th>3.1</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (x)</td>
<td>1.1311</td>
<td>1.1632</td>
</tr>
</tbody>
</table>

Then \( f (3.16) = \)
\[ a- 1.1334 \quad b-1.552 \quad c-1.1505 \quad d-1.1515 \]

4- If you have the equation \( x \ln(x) - 1 = 0 \) with root in the interval \( [1, 2] \), then if you use the bisection method then the interval which contain the root in the second iteration is
\[ a- [1.2, 2] \quad b- [1.5, 2] \quad c- [1.75, 2] \quad d-[1,1.75] \]

Q3 Solve one of the following

1- What is the number of iteration which you need to find the approximation root with any \( \varepsilon \) by use Bisection method.

2- Use the Newton-Raphson Method to find the general term of \( \sqrt[3]{a} \).