Boolean Algebra laws

1. $A+B=B+A$ $A*B=B*A$ **(Communicative Property)**
2. $A+(B+C)=(A+B)+C$ $A*(B*C)=(A*B)*C$ **(Associative Property)**
3. $A*(B+C)=A*B+A*C$ **(Distributive Property)**
4. $\overline{A+B}=\overline{A}*\overline{B}$ **(DeMorgan's Law)**
5. $\overline{A*B}=\overline{A}+\overline{B}$ **(DeMorgan's Law)**
6. $A+0=A$ $A*0=0$
7. $A+1=1$ $A*1=A$
8. $A+\overline{A}=1$ $A*\overline{A}=0$
9. $A+A=A$ $A*A=A$
10. $\overline{\overline{A}}=A$
11. $A+\overline{A}*B=A+B$
12. $(A+B)*(A+C)=A+B*C$
13. $(A+B)*(C+D)=A*C+A*D+B*C+B*D$
14. $A*(A+B)=A$
15. $A\oplus B=A*\overline{B}+\overline{A}*B$
16. $\overline{A\oplus B}=\overline{A}\oplus B=A\oplus\overline{B}$

To prove ($A+\bar{A}*B=A+B$) can use truth table

A	B	$\bar{A}$	$\bar{A}$*B	A+($\bar{A}$*B)	A+B
1	1	0	0	1	1
1	0	0	0	1	1
0	1	1	1	1	1
0	0	1	0	0	0

To prove ($(A+B)*(A+C)=A+B*C$) can use truth table

A	B	C	A+B	A+C	(A+B)*(A+C)	B*C	A+B*C
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	0	0	1	1	1	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	0	0
0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0

To prove ($A*(A+B)=A$)

= A*(A+B)

= A.A+A.B

=A + A.B

= A(1+B)

=A(1)   =A

To prove ($(A+B)*(C+D) = A*C + A*D + B*C + B*D$) can use truth table

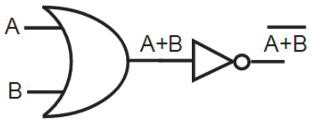
A	B	C	D	A+B	C+D	(A+B)*(B+D)	A*C	A*D	B*C	B*D	RESULT
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	0	1	0	1
1	1	0	1	1	1	1	0	1	0	1	1
1	1	0	0	1	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	1	0	0	1
1	0	1	0	1	1	1	1	0	0	0	1
1	0	0	1	1	1	1	0	1	0	0	1
1	0	0	0	1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	0	1	1	1
0	1	1	0	1	1	1	0	0	1	0	1
0	1	0	1	1	1	1	0	0	0	1	1
0	1	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	1	0	0	0	0	0	0
0	0	1	0	0	1	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

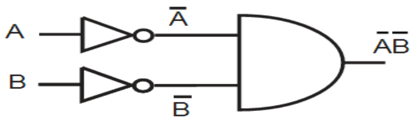
To prove ($\overline{A\oplus B} = \overline{A}\oplus B = A\oplus\overline{B}$) can use truth table

A	B	$\overline{A}$	$\overline{B}$	$A\oplus B$	$\overline{A\oplus B}$	$\overline{A}\oplus B$	$A\oplus\overline{B}$
1	1	0	0	0	1	1	1
1	0	0	1	1	0	0	0
0	1	1	0	1	0	0	0
0	0	1	1	0	1	1	1

DE MORGAN'S THEOREMS

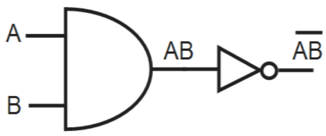
$$\overline{A+B} = \overline{A} * \overline{B}$$

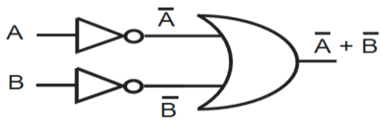




A	B	$\overline{A}$	$\overline{B}$	$\overline{(A+B)}$	$\overline{A}*\overline{B}$
1	1	0	0	0	0
1	0	0	1	0	0
0	1	1	0	0	0
0	0	1	1	1	1

$$\overline{A*B} = \overline{A} + \overline{B}$$





A	B	$\overline{A}$	$\overline{B}$	$\overline{(A*B)}$	$\overline{A}+\overline{B}$
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	1	1	1	1

Ex1: Simplify the following expression as much as possible:

$$\overline{\overline{A(A+B)} + B\bullet\overline{A}}$$

$$\overline{\overline{A(A+B)} + B\overline{A}} = \overline{\overline{A(A+B)}} \bullet \overline{B\bullet\overline{A}}$$

$$= A(A+B)\bullet(\overline{B}+A)$$

$$= (A+AB)\bullet(\overline{B}+A)$$

$$= A(1+B)\bullet(\overline{B}+A)$$

$$= A(1)(\overline{B}+A)$$

$$= A(\overline{B}+A)$$

$$= A\overline{B} + A\bullet A$$

$$= A\overline{B} + A$$

$$= A(\overline{B}+1)$$

$$= A(1) = A$$

<u>TO PROVE</u>

A	B	$\overline{A}$	(A+B)	A.(A+B)	$\overline{A.(A+B)}$	B.$\overline{A}$	A.(A+B)+ B.A	RESULT
1	1	0	1	1	0	0	0	1
1	0	0	1	1	0	0	0	1
0	1	1	1	0	1	1	1	0
0	0	1	0	0	1	0	1	0

$$\overline{A(A+B)} + B\overline{A}$$

$$\overline{\overline{A(A+B)} + B\overline{A}}$$

$$\overline{\overline{A(A+B)} + B\overline{A}} = A$$

Ex2: **Simplify the following expression as much as possible:**

$$\overline{\overline{A+B} + \overline{A}*B}$$

$$\overline{\overline{A+B} + \overline{A}*B} = \left(\overline{\overline{A+B}}\right)\left(\overline{\overline{A}B}\right)$$

$$= \left(A+B\right)\left(A+\overline{B}\right)$$

$$= AA + A\overline{B} + BA + B\overline{B}$$

$$= A + A(B+\overline{B}) + 0$$

$$= A + A(1) = A + A = A$$

Ex3: **Simplify the following expression as much as possible:**

$$(\overline{\overline{A}+\overline{B}} * \overline{C}) + (A * \overline{(B+\overline{C})})$$

$$\overline{\overline{A}+\overline{B}} * \overline{C}) + \left(A * \overline{(B+\overline{C})}\right)$$

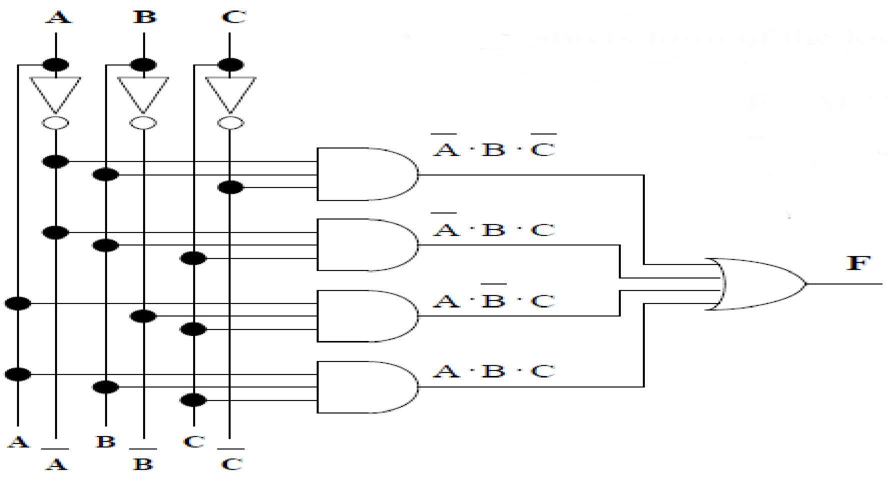
$$= \left(A*B*\overline{C}\right) + (A*\overline{B}*C)$$

$$= A*(B*\overline{C} + \overline{B}*C)$$

$$= A*(B \oplus C)$$

Ex4: **Simplify the following expression and draw the Boolean expression :**

$$\mathbf{F} = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$



$F = (\bar{A}B\bar{C} + \bar{A}BC) + (A\bar{B}C + ABC)$

$\quad = \bar{A}(B\bar{C} + BC) + A(\bar{B}C + BC)$

$\quad = \bar{A}B(C + C) + AC(\bar{B} + B)$

$\quad = \mathbf{\bar{A}B(1) + AC}(1)$

$\quad = \mathbf{\bar{A}.B + A.C}$